STRENGTH

AND

CALCULATION OF DIMENSIONS

OF

IRON AND STEEL CONSTRUCTIONS,

WITH

REFERENCE TO THE LATEST EXPERIMENTS.

TRANSLATED FROM THE GERMAN

OF

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PREFACE.

Many experiments have been lately made in Germany, England, Sweden, and America, to determine the properties of iron and steel. We propose to give in this brochure a general view of the results obtained, and of their consequences, without much detail, but so complete as to place practical engineers at the present stand-point of critical judgment.

These experiments show (what every one admits) that the methods hitherto employed in calculating the dimensions of iron and steel constructions have been entirely wrong; and that the security of structures, in which their results have been applied, though with great expenditure of material, is much less than supposed.

Several methods for attaining better results have been projected; one of which was adopted by the Bavarian Government. The brief sketch of the several methods, given in the Appendix, shows that Launhardt's deserves the preference. This is so obvious, and meets with so few objections, that it is unsatisfactory only because of its limited application. A formula like that of Launhardt was needed for the case of resistance to alternating tenison and compression. Such a formula is here deduced. With it we have all the requisites for a simple and rational determination of dimensions. It is to be hoped that no one will wait to consider it until more bridges are destroyed.

The chief reason that no one of the new methods has been generally employed is, that no one of them is complete. It would be impossible to determine fully the dimensions of a bridge by the use of any one of them, except by the addition

of deductions specially made in each case. For this the working engineer has no time.

The systematic and final investigation here presented also includes the cases, so far unconsidered, which occur under shearing stress. Very particular regard is given to the subject of rivet-connections.

The ordinary methods of static calculation are not changed by the new method. Those who prefer graphic solutions will find all that is necessary for the complete determination of stresses after completion of the diagram of forces.

The new formulas are based upon Wöhler's law; but the special results of his tests must be applied with judgment;—no more reliance being placed upon them than upon those of Rondelet or Brunel under the old methods. General formulas, old or new, do not change because of new experiments.

In the calculations especial reference is had to bridge and building constructions, in which permanent duration is required. Consideration of the special resistances and experience will serve to determine the co-efficients of safety.

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ERRATA.

Page 17, in heading, for *proof*, read *ultimate*.

" 18, line 11, after [illegible], insert [illegible]

" 21, line 3, for *bracing*, read *flange*.

" 22, line 22, after [illegible] substitute [illegible] only $\frac{1}{10}$*th*.

" 34, line 13, insert *to* at beginning.

" 42, line 3, for *struts*, read *trusses*.

" " 2d line from bottom, before *influence* insert *no*.

" 51, lines 10, 13, 22, for *girder*, read *flange*.

" 52, 4th line from bottom, for *with equal*, read *at*.

GENERAL PROPERTIES. DIMENSIONS.

Until within a short time the dimensions in steel and iron constructions were determined in the following way: The maximum strain, B, to which a member of a structure could be subjected, was found, and then divided by the permissible strain on the surface unit

$$(1)\ F = \frac{max.\, B}{b}$$

which gave the area in superficial units of the section required for the member. The same value was always given to b, both in case of constant and variable strains. In Prussia, for example, it was generally assumed that for iron, $b = 730$ kil. per sq. cm.; and this served for tension, compression, and shearing.

Gerber made a new departure in the case of the Mayence bridge. A different b was taken for each member, varying inversely as the ratio of the strain due to total load to that due to weight of bridge.

Again, if a bar were subject to alternate tension and compression, the same formula was employed; $max.\, B$, indicating the greatest *absolute* value of B. The Americans were wiser, for they used the formula

$$F = \frac{max.\, B + max.\, B'}{b}$$

in which $max.\, B'$ is the greatest strain in the sense opposite to that of B.

Numerous breakings of axles, boiler explosions, and failures of bridges, repeatedly called attention to the causes of these phenomena. Safety co-efficients were always introduced, which seemed to preclude all danger. Still the question, whether our iron bridges in general will live out their assigned terms, forced itself into notice. Experience can give no answer, for the use of iron in bridge-building dates back hardly a century. In 1874, the Union of German Architects and Engineers determined to seek a solution, by systematic observations. These observations are of the greatest importance; but, of course, no decisive result can be reached within a few years. Meanwhile, it is well to consider the results already obtained. To the question, whether the common method of determining dimensions will stand the test of unprejudiced criticism, we shall find a negative answer. This settled, and the method consigned to the limbo of past errors, we shall consider the best guides to further investigation, as suggested by the results of theory and practice brought down to date. "In order to see aright, one must know where to look," as Schelling says.

§ 1.

Wöhler's Law.

The experiments upon which the methods hitherto employed depended have been made during the course of a century by Perronet, Poleni, Telford, Brunel, and many others. Many of these experiments were very carefully made, and are not worthless; but they were all based upon a partial view. It was thought that a body once subjected to a certain strain, and withstanding it, must be able to endure the same strain, no matter how often repeated.

Proof was made by gradually increasing load of the single pull, pressure or shear, just sufficient to break a bar of square unit section; and the number, t, so obtained, was regarded as

the corresponding strength of the material. This t is called the ultimate strength; and we know that any strain, whether constant or gradually increasing, but always less than t, will not rupture the material by a single application.

That violent and frequent shocks are especially unfavorable in their effects has always been known; but, in 1858, A. Wöhler showed that besides this, as a basis of trustworthy calculation, experiments concerning resistance to often repeated strains must be made. Fairbairn immediately made trial of a riveted girder; first loading it with $\frac{1}{4}t$, then with $\frac{1}{3}t$. It stood 1,000,000 strains with $\frac{1}{4}t$, and broke with 313,000 more strains with $\frac{1}{3}t$. But general conclusions cannot be drawn from those results; for the apparatus was so contrived that the effects due to load, and those due to other disturbing causes, could not be distinguished.

In the years 1859 and 1870, Wöhler made very exact and comprehensive experiments on iron and steel. The test-bars were made specially for the purpose, and all disturbing influences were eliminated. It was found, as was expected, that while a certain strain t, once applied, may rupture the material, a less strain, often repeated, will also induce rupture. Here was a new point of observation reached. It was obvious that the change in the grouping of molecules, caused by the changing strain, affected the resistance of the material unfavorably. Hence ease of rupture must be directly proportional to the increase of difference in strains; since there was a corresponding increase in the changes of positions of the molecules. Wöhler was therefore able to state a general principle, which may be expressed as follows:—

Rupture is caused not only by a dead load exceeding the ultimate strength, but also by often repeated strains, no one of which is as high as the ultimate strength. The differences of strains are therefore effective cause of destruction of continuity in the degree that the minimum strain sufficient for rupture diminishes as these differences increase.

If the material is ruptured by the strain t once applied, strains less than t may cause breaking by repeated application; and the less the strain, the greater the number required for destruction, and conversely. Hence it is important in the determination of the degree of security to consider whether a structure is to remain in use for a limited time, as in the case of rails, axles, &c.; or is to stand for an indefinite period, as in the case of bridges, buildings, &c.

Wöhler's experiments include tension, compression and torsion. Resistance to torsion is regarded as a kind of shearing resistance, and it is assumed that the shearing forces do not lie in a plane (Fig. 1). Though the results of repeated compression were not found, it is to be inferred that they would be analagous with those obtained for tension. Not so, when compression and tension alternate. Here a single case was investigated, viz.: when the strains in both directions are equal; other cases are not yet filled out.

When Wöhler left public office (1870), he asked the Prussian Minister of Trade and Commerce to have his experiments continued, and, upon the nomination of Reuleaux, Prof. Spangenberg was commissioned to the work. His experiments during a period of three years (Wöhler's lasted twelve), are quite limited; but Wöhler's law is fully confirmed by them. Spangenberg has given his attention to other metals; and, especially to the conditions of the surfaces of fracture under different kinds of strain; attempting to explain them by a hypothesis concerning the molecular constitution of metals. Further investigation in this direction would be of import to theory and practice, since there has hitherto been a total want of any general principles to determine judgment upon questions concerning the properties of resistance.

§ 2.

Remarks upon Wöhler's Law.

Wöhler's law, as given above in general form, is doubtless correct; and it may be regarded as already established by experience, since we have often made unconscious use of it. If one wishes to break a rod with his hands, and a single effort is not sufficient, he lets it go, and gives another pull; and if this does not avail, he succeeds, perhaps, by bending it back and forth. The force of the arm is not greater in the last case; indeed, he does not need to use as much force. So it was known long ago that when there are repeated stresses in opposite directions, so that the differences of stress are the greatest, the force necessary for rupture is less than in case of stresses in a determined direction, or for a single stress.

It is surprising that for so long a time regard has not been given to the number and the kinds of strains that occur in the most important structures. Yet it is not to be forgotten that the methods of Gerber and the American engineers, mentioned above, were prompted by a correct feeling. Had more attention been given to them, it is possible that a course of experiments for years would not still be necessary to give a general but provisional expression to a law continually applied by every layman.

There is still room enough for the precise determination of Wöhler's law in its theoretic and practical aspects. In his experiments the stresses followed one another in rapid succession; but they require a certain duration of time to attain their full intensity; unless the effect of shocks proper is under consideration. What effect have the rapidity of succession, the degree of increase, and the duration of stress? The influence of the two latter upon t is not yet determined.

It is not necessary to adopt Wöhler's opinion that the different kinds of resisting strength of iron and steel can be ob-

tained from one of the metals. It is enough to know that for stresses of determinate kind and determinate position of the plane of forces, Wöhler's law holds true.

Again, the general expression of the law and the results of experiment are to be considered separately. Of course, the figures fit exactly only those kinds of metal upon which Wöhler made his experiments. But there has hitherto been no hesitation in ascribing to material employed a resisting strength determined upon other kinds of material, although, even within the range of fixed kinds, *e. g.*, rolled iron and plate iron, differences in resistance to single stationary load, amounting to 30, 40 and 50 per cent. are common. A little while ago, had any one ventured objection, the answer would have been, that there were co-efficients of safety. But these are still employed.

Though there are some effects to be determined and a very great number of data is desirable; still we have definitely, in Wöhler's law, and provisionally in his tables, the best starting point for a rational determination of the dimensions of steel and iron members. The difference between the new and old methods is that while the former is of necessity not absolutely exact, the latter is in any event false.

§ 3.

Launhardt's Formula.

Suppose a rod of square-inch section strained but once by the ultimate load t; it will break. Make the stress a little less than t, then by Wöhler's law, a certain number of repetitions are necessary to produce rupture. Let the stress decrease, then the number of repetitions required increases. A number must be reached at which the rod is safe as against any number of stresses to which it is actually subjected. Let the stress, for the case in which the rod returns to a perfectly strainless condition, be denoted by u; and let it receive the name

given by Launhardt, original strength (Ursprungsfestigkeit). This is inversely as the number of stresses to be borne; so that for a rail which is to be changed for another in time, it is greater than for a member of a bridge which is to be permanent. We shall consider only the latter case, but the general formula will hold for all others; and u will vary between this value and the value t of ultimate strength. It follows from the definition that the difference of stress $d = u - 0 = u$.

Generally the rod does not return to a perfectly strainless condition, but there remains a minimum strain c. The stress, which in this more general case, causes fracture, Launhardt calls working resistance (Arbeitsfestigkeit), and indicates by a. The difference of stress is $d = a - c$, and $a = c + d$ (2).

By Wöhler's law, a decreases as d increases. The limiting values of a are by (2), and the definitions of u and t

$$\text{for } c = o, \quad a = d = u,$$
$$\text{for } d = o, \quad a = c = t.$$

Ultimate strength and original strength are special cases of working strength. As a is a function of d we can assume

$$a = \alpha\, d \qquad (3)$$

in which α is an unknown quantity. But we know that

$$\text{for } d = o, \quad \text{since } a = t, \quad \alpha = \infty,$$
$$\text{for } d = u, \quad \text{since } a = d, \quad \alpha = 1.$$

To these conditions corresponds the value chosen for α, by Launhardt.

$$\alpha = \frac{t - u}{t - a}$$

which remains to be tested for intermediate values by the results of experiments

From (2) $\qquad a = \dfrac{t - u}{t - a}\, d = \dfrac{t - u}{t - a}\,(a - c).$

$$\therefore\ a = u\left(1 + \frac{t - u}{u} \cdot \frac{c}{a}\right). \qquad (4)$$

Denoting by B, the stress upon a member,

$$\frac{c}{a} = \frac{\text{min. } B}{\text{max. } B},$$

hence
$$a = u\left(1 + \frac{t-u}{u} \cdot \frac{\text{min. } B}{\text{max. } B}\right) \qquad \text{(I)}$$

This is Launhardt's formula, and is applicable whenever a piece is always under the same kind of stress, whether of tension or compression. The value of u for compression is not yet determined, and the same values of t and u will be used both for tension and compression; this is justified by certain observations, and was used in respect to t in previous methods of calculation.

We shall, therefore, include the terms tensile, compressive, and shearing strength in one, and regard the working resistance as equivalent to the special stress under consideration.

It is yet to be determined whether Launhardt's choice of co-efficient a holds for intermediate conditions.

From (4)
$$a = \frac{u}{2} + \sqrt{\left(\frac{u}{2}\right)^2 + c\,(t-u)},$$

the positive sign being taken, because a is positive and greater than u. The value of t as well as of u may vary with the kind of stress and material; and a varies for a fixed value of c; hence all results should be obtained from experiments of the same kind, and with like material. The best results for comparison are, without doubt, those which Wöhler obtained with Krupp's spring cast-steel not hardened; and Launhardt's formula receives confirmation from the fact that it corresponds exactly with these results. Wöhler found for this steel, in

bending tests $t = 1{,}100$ centner, $u = 500$ centner per square inch, hence the working resistence per square inch,

$$a = 250 + \sqrt{62500 + 600\,c}.$$

This equation gives the values in the third line below; Wöhler's results appear in the second.

For $c =$	0	250	400	600	1,100
a, by experiment, $=$	500	700	800	900	1,100
a, by Launhardt, $=$	500	711	800	900	1,100

By former hypothesis, only the stress of 1,100 would have made rupture possible; while we see in the table that all stresses, down to 500, were sufficient to cause rupture.

§ 4.

Formulas for Alternating Tension and Compression

It often happens that the same member is subjected to alternate compression and tension. Since Launhardt's formula cannot be applied, another will be obtained by like reasoning, dependent upon Wöhler's Law. Wöhler has investigated the important case in which the stresses in both directions are the same, calling the resistance (s) vibration-resistance. If the strain in one direction is zero, then the resistance is denoted by u, the original resistance. Two limiting cases are given.

Let a rod of square-unit section be subjected to alternate tension and compression. To each value of a of the greater of these stresses corresponds a certain value a' of the smaller in this respect; that for the greatest number of vibrations between $\pm\, a$ and $\mp\, a'$ the material remains sound. The difference in stress $d = a + a'$, therefore

$$a = d - a'. \qquad (6)$$

According to Wöhler's law, a varies inversely as d. Assume

$$a = \alpha\, d. \qquad (7)$$

But, for $a' = o$, $\quad a = u = d$,
for $a' = s$, $\quad a = s = \frac{1}{2} d$.

Hence from (7),

for $a = u$, $\quad \alpha = 1$,
for $a = s$, $\quad \alpha = \frac{1}{2}$.

These conditions give the co-efficient

$$\alpha = \frac{u - s}{2\,u - s - a.}$$

Hence from (6)

$$a = \frac{u - s}{2\,u - s - a} d = \frac{u - s}{2\,u - s - a} (a + a'),$$

and hence $\quad a = u \left(1 - \frac{u - s}{u} \cdot \frac{a'}{a}\right) \qquad (8)$

Now, if for a given member in a structure, max. B is the greatest stress exerted, whether of compression or tension, and max. B' the greatest in the opposite sense, we have,

$$\frac{a'}{a} = \frac{\text{max. } B'}{\text{max } B},$$

$$\therefore a = u \left(1 - \frac{u - s}{u} \cdot \frac{\text{max. } B'}{\text{max. } B}\right), \qquad \text{(II)}$$

and the value of a denotes the working strength.

The original resistance and the working resistance in the direction of the greatest absolute stress, max B, are denoted by u and a. As u for compression is not yet known, the value for tension may be provisionally employed, being somewhat too small.

In some constructions the oscillations between a and a' begin with a stress equal to zero; in others, with a stress equal to c., mostly caused by the dead weight. The operation of a complete forward and back vibration must be the same, and cannot be essentially changed by the longer action of c, which lies far within the limits of elasticity.

Formulas (I) and (II) serve not only for stresses by tension and compression, but also for all other kinds, if the values of t, u and s are known.

If Φ denotes the ratio of the limiting stresses, the least to the greatest, on a member of a structure, our formulas read

For stress in a determined direction:

$$a = \left(1 + \frac{t-u}{u} \Phi\right) \qquad \text{(I } a.\text{)}$$

For stress in the opposite direction,

$$a = u\left(1 - \frac{u-s}{u} \Phi\right) \qquad \text{(II } a.\text{)}$$

§ 5.

Proof Strength for Tension and Compression.

The old experiments with wrought-iron give more uniform and higher figures for ultimate strength than the later. Navier gives the results of seven experiments in France, England and Italy; the mean, per square centimeter, being 3,940, 4,220, 4,290, 4,450, 4,610, 4,680, 5,010 kil.

Under conditions otherwise equal ultimate strength is dependent on the working of the metal. Kirkaldy found for round and square iron, as a mean of many trials, 4,050 (variations from 3,780 to 4,330); Wöhler, for Borsig and Kömgshütte round iron 4,110 (from 3,730 to 4,530); Knutt Styffe, soft puddled iron, 3,400 for round iron and 3,460 for square iron.

From 17 trials of English rolled iron at 3 shops, Styffe obtained 3910, (from 2,940 to 5,100); from 16 with Swedish rolled iron, at 4 shops, 3,760 (from 3170 to 4,900). Bauschinger obtained for Wasseralfingen rolled iron, 3,890 (from 3,750 to 4,140); for angle-iron at the Lothring works of 6 by 6 and 7 by 7 cent'r, 3,195. Kirkaldy's mean for angle-iron (many experiments) was 3,850 (from 2,910 to 4,310).

For Borsig rivet-iron, Wöhler found from two trials 5,120; for English Homogeneous iron, 3 trials, 4,280. A piece from the head of an English rail gave to Styffe as average of 3 tests 3,380; another piece, with 2 trials, 3,090; and a piece from Low Moor tire-iron 3,760. Bauschinger got for gas pipe perpendicular to direction of rolling, 1,400—1,500.

Styffe puts the strength of soft iron for tension at 3,380; Gerber and many others assign 3,500 for bridge construction; Reuleaux assigns 4,000; Von Kaven deduces from Kirkaldy's experiments for wrought-iron the average value, 4,200. For good iron, suitable for bridges, the ultimate tension must lie between 3,500 and 4,000, (see § 12). Rolled figured-iron generally has little proof-strength and little tenacity; its use should be avoided as much as possible.

For iron wire suitable for bridge construction, Navier deduced from the experiments of Buffon, Telford and Seguin the averages 6,000, 6,360, 6,000; Mosely considered 6,580 as permissible, Reuleaux 7,000, Von Kaven (from Kirkaldy's results) 6,700; Laissle and Schübler, 5,000 to 8,000: 6,000 may be taken as a mean; but tests are always in order. The ultimate resistance to tension in plate-iron is generally less than for other sorts, and there is often a marked difference depending on the direction of stress. The value is generally greater for longitudinal than for transverse stress. Like relations appear in the kinds of iron used in bridges; but as the stress is generally only longitudinal, the matter is of less interest.

Kirkaldy obtained from a great number of plates, length-

wise, 3,570 (from 3,210 to 3,870), and transversely, 3,250 (from 2,920 to 3,550). On the other hand, Fairbairn, from four kinds of boiler iron, found 3,540 lengthwise (from 3,089 to 4,060); 3,620 across (from 2,940 to 4,330). From several boiler plates, Bauschinger obtained from twelve experiments, longitudinally, 2,820 (from 2,600 to 3,270); transversely, 2,730 (from 2,350 to 3,180). Boiler plate from the exploded locomotive "Fugger," gave in undamaged places, lengthwise, 3,040; across, 2,880. Stevens, in America, with the best Low Moor boiler plate, obtained, as a mean of five trials, lengthwise, 4,140 (from 3,890 to 4,500); and with cistern plate, a a mean of six tests, 2,900 (from 2,320 to 3,670). Bauschinger obtained from a piece of decided fibrous texture, 2,910 along the length, 1,910 across. In tests at Gonin & Co., Paris, the longitudinal strength was greater than the transverse; but for charcoal-iron in section only $\frac{1}{45}$, and for coke-iron $\frac{1}{5}$.

From Kirkaldy's experiments, Von Kaven obtains a mean of 3,890 for plate-iron. The English Admiralty requires for first quality 3,460 longitudinal, 2,830 transverse; for second quality, 3,150 and 2,680 respectively, warm and cold bending tests being required. Without special experiment the stresses should not exceed 3,000 longitudinal and 2,700 transverse. The ratio $\frac{9}{10}$, transverse to longitudinal, agrees well with Kirkaldy's mean and with the tests of Edwin Clark.

In the case of steel, the ultimate tensile strength depends largely upon the quantity of carbon and other ingredients; we shall return to this in another place. As the quantity of carbon is not always known, general results only can be given. Kirkaldy obtained as a mean of 9 different kinds, 6,770, from 4,930 for puddled steel up to 9,340 for cast-steel. Sheffield Bessemer gave 7,840. Wohler found for cast axle-steel from Krupp, Borsig, Vickers and Bochum, an average of 6,250, with eleven tests; from 4,020, for Vickers, to 7,670 for Krupp. Again, for heads of Krupp cast-steel rails, 7,380; for Frith tool steel, 8,400. In the case of hammered Bessemer round

steel of from 0.86 to 1.35 per cent. carbon, Styffe found a mean of 7,730 (from 6,880 to 8,970), with eight tests; again, from rolled Bessemer steel, square and round, of 0.38 to 1.39 per cent. carbon, a mean of 6,180 (4,550 to 9,840), with nine tests; and for rolled Swedish round cast-steel of from 0.69 to 1.22 per cent. carbon, a mean of 8,910 (7,280 to 10,170), with four tests. We may assume for puddled steel 5,000; for good medium hard Bessemer steel, 5,500 to 6,500; for very good and hard cast-steel, 8,000. The last value is given by Reuleaux, Laissle, Schübler, and others.

For Styrian cast steel plate (Bessemer) Bauschinger found as the mean of two tests 5,025 longitudinal and 5,180 transverse, Wöhler, in five tests on Krupp's cast-plate-steel, found an average of 5,390 long. (from 4,900 to 5,770) and for that of Borsig 5,040 in one test. Tresca obtained 5,400 and 5,760 long. in two kinds of plate cast-steel; Stevens, with six tests on best English Bessemer steel 5,880 (5,240 to 6,090). For plate steel, longitudinal and transverse, 5,000 may be assured.

For the ultimate resistance to compression we have no experiments. It is hard to define it in a way practically sufficient. Bauschinger, in experiments on steel, found that a complete destruction of the material was hardly to be accomplished by compression, and he was of opinion with Rondelet, that metal yields sooner by bending than by crushing whenever the depth is more than three times the least transverse dimension (§ 9). Rondelet, and after him Navier, put ultimate strength for compression at 4,950, Moseley at 6,580; and Bauschinger found the resistance of Bessemer steel considerably greater for compression than for tension. Though in Wöhler's and Spangenberg's experiments the fracture always first occurred on the tension side, it does not necessarily follow that the metal yields to one strain more than to the other; and it is safe to assume an equality of working-resistance for tension and compression. But it is assumed that crushing of the compressed parts is not to be feared. Fairbairn, in several

tests with compound plate-beams, observed that the fracture began in the upper flange; since that time care has been taken to stiffen as required, and to provide a rigid bracing at that point.

§ 6.

Excess of Elastic Limit.

The limit of elasticity is generally defined as that stress per square unit beyond which permanent changes of form occur, while under less stresses the body returns to its former condition. Reference is made, not to sudden changes in stress and shocks, but to gradually increasing strains. But the definition is theoretically worthless, for a limit so definite is not probable, and much less is it proven. On the contrary, Hodgkinson and Clark have observed that there are permanent changes of form under very small loads. At present we must be content with defining this limit with Fairbairn, as that stress below which the changes in form are approximately proportional to the forces, while above this they increase much more rapidly. The words "approximately" and "much" are not so indeterminate as might be supposed, for, in the experiments of Bauschinger, the passage beyond the limit of elasticity could be determined very precisely; as for example in tension; "for with the same increase of load a disproportionately great elongation occurred at once, the maximum of which was in every case reached after some time." This sudden elongation must be credited to permanent changes of form; further elongations until near the breaking limit remain proportional to the stresses, and the modulus of elasticity is always found to be independent of the latter. (§ 9.) In the first definition the changes of form which are permanent from Bauschinger's point of view are neglected. All experiments, up to the present time, have shown that when the elastic limit is passed, the tensile resistance is con-

siderably increased, while ductility and tenacity diminish; the metal becoming brittle, and having little power of resistance to shock. In experiments at the Woolwich Arsenal, an iron rod, four times ruptured by pull, gave the successive values of t: 3,520, 3,803, 3,978, 4,186; Bauschinger tore apart a piece of iron seven times, and the resistance increased from 3,200 to 4,400.

Paget found that iron chains after stretching bore a greater dead weight, but had less resistance to shock. Fairbairn thought all these phenomena could be explained by the hypothesis that the resistance of all the parts was not at first called into action, but, like ropes, they became gradually strained in common under sufficient load. With this accords the fact that Bauschinger observed that increase of resistance, especially in rolled iron, was notably regular when the stress was in the direction of the fibres. The analogy holds further; for a rope, when tense, is more easily broken by shock. And this explains why a rod under sudden increase of stress breaks more readily than in case of gradually increasing pull.

When the limit of elasticity is passed, this limit is again raised. Tresca, in tests of rails, succeeded in pushing the limit of elasticity to near the limit of rupture, so that it was less by about one-tenth. The practice hitherto has been to assume as permissible stress (b) a fraction of the elastic limit. In this case b increases with the number of loads. But the material becomes more brittle, and less resistant to shock, and local passages beyond elastic limits are not excluded. So that we need not assent to the often-advocated opinion that a test of material beyond the elastic limit would be of advantage. It is worth mention that the increase of resistance with the passage beyond each limit cannot go on indefinitely; but a diminution must occur at some time, unless we assume that with very gradual increase of stresses and longer intervals, the original resistance becomes greater than the initial ultimate strength.

Now, if passage beyond the elastic limit can work unfavorably, it should not be permitted. But it is enough to know that, according to the numerous experiments of Styffe and others upon all sorts of iron and steel, the ratio of elastic limit to ultimate strength generally lies between $\frac{1}{1 \cdot 4}$ and $\frac{1}{1 \cdot 8}$, and under the most unfavorable circumstances seldom reaches $\frac{1}{2}$.

Wertheim and Styffe have attempted to establish more precise definitions of the elastic limit, but as they are not better, either theoretically or practically, than others, it would be superfluous to consider them. It is since the time of Hodgkinson and Clark that an empirical importance has attached to this limit; and it is still very narrow in its scope, because the limit, as above defined, is of no avail in case of sudden change of strain and of repeated stresses.

Vicat made experiments to determine the effect of lapse of time upon a dead load. He kept wires loaded up to $\frac{3}{4}$ths the tensile resistance, during thirty-three months. The one with heaviest load broke. Vicat inferred from this, and because the extension seemed to be proportional to the time, that every load beyond the elastic limit would, after lapse of time, cause rupture. Considering that very small loads cause permanent changes in form, it would be more correct to infer that any load, if given time enough, will cause rupture. Fairbairn thought he could prove this by tests on cast-iron girders. But we do not find that the results of his experiments warrant his conclusion. But the fact that under stress beyond the elastic limit the ultimate strength increases, leads to the conclusion that security against dead-load increases with time. But if it is objected that a decrease may follow an increase of ultimate strength, it must be admitted, in view of all that has been said, that the influence of duration of dead-load has not been clearly determined. That each load requires a certain time to cause its correspondent permanent change has been known since the time of Hodgkinson and Wertheim, and also accords with Fairbairn's comparison with

ropes; and, again, it has been observed by Bauschinger. This also holds true for further changes in form; and if a rod stretched again when released, does not at once return to its previous condition, a so-called secondary action takes place. This was observed in Kupffer's experiments. Thurston thinks that in this he has discovered a new phenomenon; that ultimate strength and elastic limit increase after a strain greater than the latter, continued for twenty-four hours. But there is nothing new in it. That the tensile resistance of iron and steel is greater under the action of an electric current, and that the ductility is effected, now one way, now another, by dipping the metal in acid, seem to be shown by detached experiments, but this needs confirmation.

§ 7.

Mechanical Treatment. Heating. Hardening.

Elastic limit and ultimate strength are both increased when the limit of elasticity is exceeded; ductility and tenacity diminish. Since under rolling, hammering, and pulling the elastic limit in the affected places is certainly passed, and permanent changes in form take place, the necessary effect of such mechanical treatment is obvious.

Heating and slow cooling has an effect exactly opposite to that caused by passing the elastic limit, for the metal becomes more ductile and loses in ultimate strength. According to Tunner, the brittleness produced by mechanical treatment gradually decreases if the body is allowed to remain at rest. A wire which broke when bent to an obtuse angle, just after leaving the plate, increased in pliability within a few days, and continued to do so during some weeks.

That cold-rolling considerably raises the ultimate strength was clearly shown by Kirkaldy's experiments, t nearly doubling in value, passing from 3,220 to 6,260, while anneal-

ing reduced it to 3,580. Styffe had an iron rod, which had been previously annealed, hammered cold to half its original section; the strength was raised from 3,140 to 5,830. According to Kick, United States cold-rolled iron is much more brittle than the common sort. It has often been observed that the ultimate resistance of cold-rolled metal is diminished by removal of the skin, the effect of rolling being materially greater at the surface. These phenomena and many others, having no apparent relation to one another, are all explained upon the hypothesis mentioned.

If the mechanical treatment is with heat, both influences operate, viz.: passage beyond the elastic limit and heating. These must counter-act, entirely or partially, and the metal may gain in strength, the tenacity remaining constant or increasing. In England the working of the metal is often repeated.

A body once annealed is further changed only by higher heat, unless, meanwhile, it has received some treatment with opposite effect. It follows, that the effect of annealing must be greater in the degree that the temperature is higher than that under the previous mechanical treatment. This was observed by Styffe.

Hardening produces upon steel and wrought iron an effect like that due to passing the elastic limit, with this qualification, that in the case of steel, not only ultimate strength and elastic limit, but also brittleness are notably increased. Tempered metal is not suitable for many purposes, because of its slight power of resistance to shocks. The process of tempering consists in plunging the red hot metal into some fluid, oil or water, which suddenly cools it. Brittleness may be somewhat reduced by gradual heating, and may be destroyed by annealing, together with all other qualities due to hardening. The effect of hardening is much greater upon steel than on iron; and in either case depends upon the chemical constitution and other conditions.

Tresca, by hardening, raised the ultimate strength of two

kinds of plate steel from 5,400 to 8,784, and from 5,764 to 8,880. Wöhler cut several bars from a hardened cast-steel axle and found that the strength of one was 9,209, while that of the other, which had been annealed, was 7,455. Numerous tests of the effects of hardening have been published by Kirkaldy; with which those obtained by Styffe agree in the main. It is shown by experiments made by Wöhler, Heusinnger, Waldegg and others that metal contracts a little when hardened; Wöhler finding that the contraction of a steel-rod of 33 mm. section was about 1 mm. to a meter in length.

With respect to the strength of welds we have the results of Kirkaldy's experiments. The decrease of ultimate tensile strength varied between 2.6, and 43.8 per cent.; while ductility diminished, especially that of steel. According to Smyth, the strength of welds depends mainly upon the thorough elimination of the flux employed to hinder oxydation. Welds should not be used in important bridge-members.

A diminution of strength occurs in cutting screws, amounting, according to Kirkaldy, to from 7 to 30 per cent. The cause may be that the hard surface of the rod is removed by cutting; it may sometimes be due to the cracks made by sharp dies. This, as well as the hardening caused by the greater force applied, explains why screws cut by Kirkaldy with blunt dies held better than those cut with sharp dies. That the strength of screw-bolts of small diameter proved somewhat greater, is no cause of wonder; for Kirkaldy observed that the strength increased with diminishing diameter, which was to be expected because of the proportionally greater effect of rolling.

§ 8.

Influence of Form.

The form of a member may greatly modify its strength. The bar shown in Fig. 2 has less resistance per square inch of section than if it were limited by the dotted line. For the

load at the right of the dotted line is transmitted only by the fibres contiguous to the angle to those at the left; the former, therefore, receive more than the average stress per square unit, and fracture will take place sooner at the angle. In consequence of the bending which must take place in the case represented in the figure, the stress is increased; and the load would also act unfavorably at the smaller end (§ 18). We can now understand why Wöhler found the strength of bars with abrupt change of section less than for those with rounded fillets; for in the latter case the effect of the load was gradual. In several cases the strength in the first case was from $\frac{3}{5}$ to $\frac{4}{5}$ as great as in the second, under like conditions; but these experiments do not give permanent data, since the change of section and all the modifications mentioned in the last paragraphs must come under consideration. Experiments by Fietze have proven that the notches at the base of rails, which are intended to prevent their sliding along the track, are much more prejudicial than the ordinary theory supposes. And grooved axles, subjected to torsion, exhibit a like loss of resistance.

It is remarkable that a rod like Fig. 3, will bear a greater dead pull than if the whole rod had the smaller diameter or were grooved through a greater length. The contrary was to be expected. Vickers found that a rod with a very short groove bore 12,500 kil. per sq. cr.; while one turned down a length of 35 cm. bore only 9,440. In Kirkaldy's experiments with rolled iron, very short-grooved rods of about 3-4 diameter in length, had an increased tensile resistance of about 1-3d.

These phenomona are hard to explain, but may, perhaps, be accounted for as follows. Each pulled bar bent under a heavy load because of the non-homogeniety of the material. The strain caused by the bending contributed to the breaking, but this was less, the shorter the turned portion. If this explanation is correct, then a very short rod must gen-

erally bear more dead pull than a longer of the same material and the same section. Whether this is the fact I do not know. Again, there must be a like difference with compression, and this has been verified by the observations of Bauschinger (§ 9), and others.

Nearly all experiments up to this time, Wöhler's included, have been made on plain bars. Fairbairn has tested riveted girders with the special purpose of comparing the values of different kinds of sections. The girders almost always gave way by the lateral buckling or crippling of the upper flange, which we now try to prevent by stiff flanges and by bracing with angle or T iron set at uniform distances. The relation of the strength of compound pieces to simple members has not been determined. But it is certain that this ratio greatly depends upon the efficacy of connections, so that more care should be taken in this respect than heretofore.

§ 9.

Percentage of Carbon, &c.

What is meant by the terms wrought-iron, steel, and cast-iron is more easily felt than explained; a definition, correct to-day, may not be so to-morrow. The latest authorities say that wrought-iron should contain about $\frac{2}{3}$, steel from $\frac{2}{3}$ to 2, and cast-iron more than 2 per cent. of carbon. But steel is to be found with $\frac{1}{4}$ and less per cent. carbon, and wrought-iron with about 1 per cent. Again, it is said that steel, but not wrought-iron, can be hardened; but steel with much phosphorus and little carbon cannot be hardened; wrought-iron, and even cast-iron, under certain conditions, may be made harder.

Greiner, Director of the Bessemer Works at Seraing, and Phillipart gave this definition of steel as contrasted with wrought-iron: "By steel is meant that kind of iron which can be obtained by fluid processes, and which, on account of its

consequent homogenity and compactness, is capable of offering a greater resistance; and which is also, because of the method of production, more uniform, both in composition and behavior." This would exclude many products from the category of steel.

Benedict's definition of cast-iron, correct in the main, is this: "By cast-iron is meant that obtained directly from ores, which does not admit of being wrought or welded; which melts at a lower temperature, and which contains the greatest proportion of carbon and foreign matter." Either one of the constituents of this definition alone is insufficient; *e. g.*, wrought-iron and steel can be got directly from the ore by Siemen's process.

Chemically pure iron has hitherto been obtained only in small quantity; it can be made very soft or very brittle, and is hard to melt. Iron becomes technically useful by combination with charcoal. This amounts to from 0.1 to 6 per cent., in part chemically combined, in part as graphite. With regard to the two sorts of metal which receive the names of wrought-iron and steel, it may be said that in either, the addition of carbon has an effect upon strength similar to that due to passing the elastic limit, or to mechanical treatment; the hardness and ultimate strength increase; while ductility and power of resistance to shock and sudden stresses beyond elastic limit diminish. This is less observable in wrought-iron, because of the influence of other substances and of the mechanical treatment. But with steel there is a limit, beyond which the ultimate strength, at least for tension and compression, diminishes; and with this the ductility, so that the properties of the metal approach those of cast-iron. The position of this limit depends upon the presence of other elements, and the influences considered in §§ 6 and 7.

Knutt Styffe thought that he had found the maximum ultimate tensile resistance of iron and puddled steel at 0.8 per cent.: of Bessemer and Uchatius steel at 1.2 per cent. The

latter agrees with the experiments of Vickers, in Sheffield, according to which the maximum is at 1.25 per cent. Karsten says that steel hardens best, and has most tensile resistance at from 1.0 to 1.5 per centage of carbon. With a greater percentage the hardness increases, but the resistance becomes less: at 1.75 per cent. all welding quality is lost; at 1.8 per cent. it works under the hammer with great difficulty; at 1.9 per cent. it can be worked no longer; and at 2 per cent. it has reached the boundaries between steel and cast iron; it cannot be drawn out at red heat without cracking and breaking under the hammer.

Bauschinger has made some very interesting tests of Ternitzer Bessemer Steel. The test-pieces were made for the purpose, and were of the same sort, but contained different proportions of spiegeleisen. The results for ultimate resistance were as follows:

K. %	Tension	Formula (9)	Compression	Shearing	Bending
0.14	4,480	4,435	4,780	3,410	7,920
0.19	4,785	4,510	5,380	3,710	8,[illegible]
0.46	5,330	5,270	6,330	3,585	8,340
0.51	5,600	5,480	7,000	4,020	9,[illegible]
0.54	5,560	5,620	6,110	3,930	8,550
0.55	5,650	5,665	6,170	4,000	8,825
0.57	5,665	5,705	6,550	3,945	9,[illegible]
0.66	6,295	6,245	6,550	4,280	8,[illegible]
0.78	6,470	6,[illegible]	7,305	4,140	8,750
0.80	7,230	7,134	9,670	4,820	7,645
0.87	7,335	7,640	8,910	5,000	7,550
0.96	8,305	8,540	9,880	5,820	8,680

The elastic limit increased from 2,950 to 4,870; 2,775 to 5,000; 3,750 to 4,425. Setting off the tensile resistances as ordinates to the percentages of carbon as abscissas (Fig. 4), a number of points marked by a cross is determined, grouped about a curve (1), of which the equation is

$$t = 4,350\,(1 + K^2) \qquad (9.)$$

in which K means the percentage of carbon. By means of this equation the values in the 3d column of the preceding table are found. In Fig. 4 the results of other tests are shown, notation as follows:

+ The results obtained by Vickers;

O By Styffe, with hammered Swed. Bess. Hogbö round steel;

□ By Styffe, with rolled Swed. Carlsdal Bess. square steel;

O *u* By Styffe, from rolled Swed. Uchatius cast-steel, round, Wykmannshyttan;

O *k*. By Styffe, with soft hammered Krupp cast axle-steel;

□ *t* Bauschinger, with rectangular tie-bars of Ternitzer Bes. steel;

O *t* By Bauschinger, with round rods of Ternitz Bessemer steel.

The figure shows that formula (9) corresponds fairly, not only with Bauschinger's results, but generally with mean ultimate tensile resistances; and that important deviations may occur from various causes. The equation

$$t = 3{,}700\ (1 + K^2) \qquad (10.)$$

which corresponds to curve II gives results, below which in general the ultimate resistance will not fall.

With respect to the results obtained for compression in the above table the following must be noted. Test rods of 3 by 3 by 9 cm. were strained between two compressed plates. With the increase of load an S-formed curvature was observed, which increased more and more, till the prism suddenly sprung out. The strain on the fibre at the moment of springing he regarded as the ultimate resistance.

Bauschinger's tested pieces were of the form shown in Fig. 5. The load was increased, and a pressure was reached under which without further increase, the prism contracted in length

to less than half, while the transverse dimensions increased. The stress per square cm. at this limit, which Bauschinger regarded as the ultimate strength, increases with the percentage of carbon, from 9,250 to 17,800. On the other hand, the elastic limit was independent of the kind of test. Generally very short steel prisms may be loaded to double the amount permitted for tension.

Phosphorus, like carbon, increases the elastic limit, and ultimate tensile resistance, but diminishes the power of resistance to blows and to stress differences. It makes iron brittle, coarsely crystalline, and "cold-short," that is easily broken under cold working. For this and other reasons it cannot be used in bridge structures. Phosphorus affects steel still more unfavorably than iron. According to Greiner, steel, with from 0.2 to 0.25 per cent. of phosphorus, has too little strength for technical purposes. Phosphor-steel is best suited for rail-heads, because it resists wear; but the percentage of carbon should be diminished to prevent brittleness.

According to Sandberg and Turner, silica has the same effects as carbon, while Haswell, in the case of steel, with a certain proportion of phosphorus, ascribes it to a partial neutralization of the bad properties due to the latter. Slag helps phosphor iron by diminishing its brittleness; but it makes it hard to work without splitting and springing. Next to phosphorus, sulphur is the most undesirable ingredient, having a like effect, except that it makes the metal particularly apt to break at red heat. Manganese, too, is a bad ingredient.

The effects of the above mixtures and others upon the strength of iron and steel are not clearly determined. Concerning their effect in the foundry information can be had from any text-book upon Metallurgy.

Whether, in a given case, steel or iron is to be preferred, depends upon considerations of resistance to special strains, of lightness, security under changes of temperature, economy, &c. In the application of steel, the proper percentage of carbon

is dependent not only on the mechanical working it is to undergo, but also upon the composition of the ores and the method of production, because the proportion of other ingredients is determined by these. So Vickers recommends, for pieces subjected to both tension and shock, 0.62 to 0.75 per cent.; Styffe, for axles of Swedish steel welded, or of one piece, 0.4 to 0.6; Greiner, for axles of Bessemer steel from Seraing, 0.3; Krupp, for locomotive and marine-engine axles, 0.5 to 0.6; for coach axles, 0.6; Greiner assigns for Seraing Bessemer steel, for chains and driving rods, 0.25 to 0.35; for tires not welded and piston-rods, 0.35 to 0.45; for steel rails, 0.4; for springs, 0.45.

§ 10.

Influence of Temperature.

The influence of different temperatures upon the strength of steel and iron is not satisfactorily explained. With respect to ultimate resistance only, because of numerous experiments, has their been a growing accord of views. For most kinds of metal, especially for iron, the ultimate strength appears to increase with the decrease of temperature below zero, but also to reach a maximum at a little above 100° C. Within a certain interval near 16° the resistance is quite constant; the beginning and the rapidity of the increase and the position of the maximum are dependent upon the conditions already considered.

Fairbairn, in tension experiments with bar iron, found, in one case, the resistance at 0° equal to, in another, 1 per cent. higher than at 60°. Thurston found in torsion experiments a decided increase of strength to — 12°. Spence, in experiments in bending cast-iron, found at − 18°, a strength greater by about 3.5 per cent. than at + 15°. At higher temperatures, Fairbairn found for bolt iron the maximum of ultimate tensile strength at 163° 41 per cent. greater than at 18°;

later experiments with bar iron put the maximum at 213°. A commission of the Franklin Institute, at Philadelphia, found the maximum strength 15 per cent. greater than its ordinary value at about 288°. Styffe has published the results of numerous experiments. See his Table VII.

Beyond the maximum the ultimate resistance decreases at first slowly, but very rapidly at red-heat. In this respect, too, the different kinds of metal behave very differently, and the diminution may possibly be the quicker and more rapid the lower the temperature of the metal when under mechanical treatment. Tensile resistance Fairbairn found to diminish from 202°, where it was about the same as at ordinary temperature, a low red heat, by about 17 per cent.; up to ordinary red heat, by about 34 per cent. Experiments at the Franklin Institute found the ultimate tensile resistance, at 575° lowered by 0.66, and at 700° by 0.33 from the ordinary value. Bauschinger observed the strength of puddled plate, transverse to the direction of rolling, to be at red heat 780 kil. (2,700 ordinary), and of rolled iron along the fibres, 750 (4,430 ordinary).

These results are of importance with respect to constructions exposed to fire. Kirchweiger, of Hanover, regards the diminution of tensile strength by heating as the cause of boiler explosions; attempting to prove at the same time that a boiler filled with water may become red-hot. Bauschinger thinks it possible that the continual variations and differences of temperature of the outer and inner surfaces may diminish the cohesion of the laminæ of the plate; the inner laminæ bearing a disproportionate share of the strain, and the shearing resistance being lessened.

A frequent theme of discussion is the influence of cold upon resistance to sudden changes of stress,—shocks in particular. It cannot be denied that more axles and wheels break in winter than in summer. Styffe maintains that rupture is often due to the fact that the parts are held fast, and, therefore, cannot yield to the contracting influence of the cold: again, for

tires, axles and rails, the effect of shocks is increased by the diminished elasticity of the ground.

Sandberg, in an appendix to the English translation of Styffe's work, maintains that these are not the principal causes of breaking. He laid iron rails upon granite supports which lay upon granite rocks, so that the elasticity of the foundations might be the same in any season. The two halves of these rails were tested by blows with a 380 kil. ball at $-12°$ in winter, and $+29°$ in summer; and it was found that at $-12°$ the rail could withstand only $\frac{1}{39}$ of what it could at $+29°$. This showed, at least, that there are some kinds of iron that are weakened by frost. Styffe had tested only under dead loads, and in this respect his results were trustworthy.

Sandberg also found this peculiar result: that Aberdare rails, which bore in summer 20 per cent. more strain than those from Creusot, in winter had 30 per cent. less strength. This could be explained on the hypothesis of a difference in constitution which affected the strength unequally. Fairbairn had already shown the unfavorable effect of phosphorus and sulphur at low temperature; and Sandberg thought it possible that different results would have been reached had the metal been free from phosphorus.

Unfortunately the chemical constitution of the rails was not determined; but it seems likely, that phosphorus, which always diminishes resistance to shock, may operate more actively at a low temperature. Its effect also increases under high heat. Styffe found that the grain of a screw-bolt of phosphor-iron was so affected, that a single blow of the hammer broke it. Steel, with increasing mixture of phosphorus, loses its capacity to undergo repeated heating without losing its peculiar properties.

In the year 1871, Joule, Fairbairn, Spence and Brockbank contributed to the Manchester Literary and Scientific Society four papers upon the influence of cold upon iron and steel. All agreed that resistance to dead load was not diminished by

cold, but considerably increased. Brockbank held it certain that cold diminishes resistance to shock; this, Joule and Fairbairn did not admit. All referred to experiments. No one will question the exactness of Joule's tests; but the test-pieces were wires, needles and nails, so that the results may not hold for larger pieces; while Fairbairn and Spence tested only under dead load. A series of observations by Brockbank confirm the results obtained by Sandberg. Rails were tested with blows; and in frosty weather they had far less strength than at ordinary temperature: a hollow cast-iron core-rod, about which a cylinder had been cast, cooled down to — $7\frac{1}{2}°$, broke square and smooth, leaving a brittle-looking surface, while the pieces were made stiff and sound again by heating. A rod of round-iron of best quality, of 38 mm. diameter, which lay a week exposed to frost and was covered with ice, broke at $4\frac{1}{2}°$ under a single blow of a hammer weighing 5.4 kil.

All authorities admit the increase of resistance to tension under great cold, though they deny that there is a diminution of power to resist shocks. This is bad reasoning. It is certain that resistance to dead load is somewhat increased by frost; and besides this, according to Styffe, the elastic limit; just as is the case under hammering, rolling, hardening, &c.; but as with all the latter, resistance to shock increases, there seems to be no reason for a contrary judgment in the first case. Styffe has proved that iron becomes stiffer with decrease of temperature; agreeing with Sandberg.

Thurston concludes from results of his experiments that phosphorus and other substances, inducing cold brittleness, may impair resistance to shock at low temperatures, which seldom occur; and that in other cases resistance to dead load, as well as to shock, is increased by cold. This would be novel, but it must first be proven. Thurston's test-machine is well adapted to the lecture-room, being convenient and cheap; but it is not suitable for scientific experiments requiring results numerically exact. The velocity, an important

element, is not regulated; the methods of measurement are much too primitive to answer to small differences due to temperature; and it is not to be taken for granted that torsion-tests are best suited to determine the properties of resistance of fibrous and laminated metals.

In a report of the Massachussetts Railroad Commissioners (1874), mentioned by Thurston, it is said, that "cold does not make iron and steel brittle and unsuitable for mechanical purposes, and that it is not the invariable rule that the most breakings occur on the coldest days." The membership of the Commission is not given, nor is it certain what kinds of metal were under consideration. Did it contain a large percentage of phosphorus? Were the rails iron or steel? It has been found in Northern climates—Canada, Sweden, and Russia—that a low steel, with $\frac{1}{3}$ to $\frac{1}{2}$ per cent. phosphorus, was affected by cold much less than iron. According to Styffe, there is no authentic case in which good steel contained more than 0.04 per cent. of phosphorus; though in one English iron rail there was 0.25 per cent., and in Dudley iron 0.35.

We draw the following conclusions from all the data at hand: (*a.*) Iron and steel, which are entirely or nearly free from all foreign materials, have neither their resistance to dead load notably increased by cold, nor their resistance to shock diminished. (*b.*) Certain elements, not exactly determined, but phosphorus certainly, very much diminish resistance to shock and sudden change of stress. (*c.*) The question cannot be definitely settled until the chemical constitution is determined. (*d.*) Statistics of results in warm and cold latitudes, in summer and winter, after long frost, on days of sudden intensity of cold, are required.

The above has reference to the immediate influence of temperature. In regard to the effect of repeated changes of temperature, Wöhler conjectures that frequent vibrations of molecules caused by heat, have the same effect in destroying cohesion as vibrations caused by external forces. Data from

observation have not been obtained. Spangenberg, after examination of the fracture surfaces, did not adopt this hypothesis. Bauschinger, after testing boiler-iron, thought it possible that the strength of the plate was weakened by long action of the fire. But this decides nothing as to the effect of repeated influences. If Wöhler's hypothesis is correct, we should recognize in change of temperature a cause of destruction, not only of metals, but also of all other solid bodies. And safety coefficients would be of no avail, for if we should make one beam twice as large as another, each half of the first would be as much affected as the whole of the second. In any case, bridges and buildings, which are subjected to only slight variations in temperature, will certainly be more likely to fail from other causes.

§ 11.

Bauschinger found the ultimate bending strength of steel; *i. e.*, the greatest fibre-tension at the instant of rupture, as given by the ordinary theory, always greater than the absolute tensile resistance, (see table in § 9). Wöhler obtained a like result for wrought iron and steel; but the original strength was not less for bending than for pull. The experiments of Bauschinger and Styffe show that the modulus of elasticity for bending may be assumed as equal to that for tension, without great error. All these results show that the common theory of bending gives results accurate enough for practice. Of especial interest in this respect are Bauschinger's tests, in which the length of the gravity axis or elastic line remained unaltered by bending, and the original plane transverse sections remained perpendicular to it, even under very strong bending stresses.

Though it is not asserted that the method of calculation for very thin-walled plate-girders is exact in every respect; yet it is as sound as that for trusses, in which hinges are supposed, but rivets used; and it is safer than the ordinary method for compound trusses.

The modulus of elasticity of steel per sq. centimeter, is, according to

Bending tests by Kupffer, 2,124,990 (cast and file steel).
Pull and bending tests by Styffe, 2,412,300 (Bessemer steel).

Tension tests by Bauschinger,		2,215,500,	Bes. steel prepared for test.
Compression tests by	"	2,591,000,	
Bending tests by	"	2,110,000,	
Crushing tests by	"	2,082,500,	(Bes. round rod).
Tension tests by	"	2,310,000,	(Bes. tires).

Bauschinger found the elastic modulus for torsion and shearing to be 862,000. From these results it follows that for steel we may assume as average

For tension, compression and crushing E, $= 2,150,000$.

For shearing and torsion $E' = \frac{2}{5} E = 860,000$.

In experiments with English tire iron, bar iron and Swedish wrought iron, Kupffer gets a mean of 2,053,070; Styffe gives for good iron, with very little phosphorus, 2,171,100; but for iron containing much phosphorus and slag, 1,930,600. The following figures are established for iron :

For tension, compression and crushing, $E = 2,000,000$.

For shearing and torsion, $E' = \frac{2}{5} E = 800,000$.

No effect of carbon upon the elastic modulus could be observed; but with Styffe and Kupffer, it seemed to increase a little with the specific gravity and with lowering of temperature. Passing the elastic limit, and working in the cold condition, were found by Tresca and Styffe to cause a decrease.

According to Kupffer, hardening of hard steel decreases the elastic modulus by about 6.5 per cent.; but on the other hand Morin ascribes to cast steel a possible increase of E by hardening, by 50 per cent. By Wertheim's theory and Kirkaldy's tests, the specific gravity is somewhat diminished, if the metal is worked cold or in any way the elastic limit is passed, while the volume does not decrease, as has often been assumed. Yet all these influences are not so great and well determined that they require or permit a general review.

In calculations, the specific gravity of wrought iron may be put at 7.6 to 7.7, that of steel 7.8.

§ 12.

The Examination of Metals.

The higher the limit of elasticity, the greater the strain which a body will bear without permanent change of form. Raise this by hammering or hardening, and the body will be restored after greater strains; hence the extended use of springs. If the ordinary elastic limit served for all kinds of load, and if we were sure that it would never be exceeded, then it would be desirable to set the limit as high as possible for any construction. But the ordinary value is not sufficient in case of shock. In our riveted bridges,for example, local excesses may occur, because of unequally distributed strains. These are less dangerous, if the material is strong enough beyond this limit, so that a gradual change of form takes place, as in the case of a uniformly distributed force over the whole section.

The more extensible and tenacious the metal, the less risk in exceeding the elastic limit. It is well known that a very ductile and tough metal best resists shocks and sudden changes in stress. We should, therefore, judge of the fitness of metal, not only by the height of the elastic limit and the ultimate resistance, but also by its ductility and tenacity. The greater

the latter qualities the greater the elongation before rupture.

When a rod is broken by a pull, there is a contraction of section at the breaking-point, beginning a little before rupture; attended by a decided elongation, which is independent of that which always occurs when the elastic limit is exceeded, and is approximately proportional to the length of the rod. As the total elongation at rupture is in part proportional to the length of the rod, in part independent of it, the ratio $\frac{\lambda}{l}$ of the total elongation to the length of the rod, can determine the ductility only in the case of rods of equal length; for the shorter the rod, the greater relatively the share of elongation at the point of rupture.

Kirkaldy, who has had the advantage of very many tests in this regard, recommends that we measure the excellence of the metal, both by its ultimate tensile resistance and by its contraction at the point of rupture. The stress at the breaking point, per square unit of the contracted part, increases with both the tension and the contraction; and the stress at this time furnishes the best means of determining the resistance. The results so obtained, arranged in order, give a trustworthy scale of values; but, if the gradation were according to ultimate strength only, very ordinary kinds might stand high in the scale. Kirkaldy found that the ultimate strength of coarse, crystalline metal, was equal to that of very tough and dense sorts.

The mechanical treatment and the method of production have their influence. So plate-iron is generally of less ultimate strength and ductility than round-iron.

The Department of Public Works, in India, has published the following table of requirements for estimate and supply, based on Kirkaldy's results. Contraction is expressed in per cent. of the original cross section.

	Class C.		Class D.		Class E.		Class F.		Class G.	
	t for Tension.	Cont.	Ten.	Cont.	Ten.	Cont.	Ten.	Cont.	Ten.	Cont.
Round and Square Iron	4,250	45	4,092	35	3,937	30	3,775	25	3,620	20
Flat Bar-Iron...........	4,092	40	3,937	30	3,775	25	3,620	20	3,466	16
Angle and T Iron......	3,937	30	3,775	22	3,620	18	3,466	15	3,300	12
Plate Longit............	3,755	20	3,620	15	3,466	12	3,300	10	3 150	8
Plate Trans.............	3,466	12	3,150	9	3,000	7	2,830	5	2,675	3
Plate; Mean.....	3,602	16	3,375	12	3,233	9.5	3,065	7.5	2,912	5.5

These figures show that we should use as much flat bar-iron as possible in our bridges for the parts under tension. Round-iron is useful in roof struts.

In America, the conditions of proposals for bridges require high figures for ultimate strength (generally from 3,900 to 4,200 kil.); and test-bars must also stretch from 10 to 15 per cent. of their length before rupture. An elastic limit of from 1,600 to 1,750 kil., and uniformity of elastic modulus are prescribed. For example, in the case of the new Ohio bridge, no deviation of more than 10 per cent. from the mean modulus of elasticity is allowed. Besides this, each piece under tensile strain is subjected to a test of twice the strain calculated for it—*i. e.*, about 1,400 kil. per square centr.—and, while under this strain, it receives a heavy blow from a hammer. It is generally thought in Europe that the Americans subject their bridges to a much greater strain than we; but, for $b = 700$, it amounts to about the same.

It is obvious that for the same ultimate strength the original strength increases with the extensibility, whether u is greater or less than the ordinary elastic limit. That the latter is possible follows from the fact that Tresca could push the elastic limit nearly up to t, and that permanent changes of form occurred below the elastic limit; and that in general the ordinary elastic limit has influence upon many kinds of stresses. It is not impossible that at some time there will be

found a sufficiently determinate relation between original strength and ultimate strength and contraction; or, between the ultimate strength and strain per square unit of the rupture-surface; or generally between u and values under dead load; so that u can be at least approximately found for each metal, and the numerical values be substituted in Launhardt's formula. And the vibration-strength s could be derived from some relation, or might be estimated. Wöhler found the rates $\frac{s}{u}$ nearly the same in metals so unlike as Phœnix iron and Krupp's cast-steel; the values being respectively $\frac{7}{12}$ and $\frac{8}{15}$. It would be desirable to make a great number of tests by bending, shock, &c., of metals for which the values of t, u, and s, have been fixed by numerous experiments. We should then have a better guide for the tests required of the manufacturers.

§ 13.

Permissible Strain.

The values of the stresses having been calculated, the working strength a gives the stress per square unit, which can be maintained without rupture, under any number of repetitions. No reference is made to influences that do not admit of systematic investigation, such as shocks due to the passing of wagons in the streets, flaws, rust, &c.

A. Wrought Iron.

Tension or Compression only.

For Phœnix axle-iron, Wöhler's tests give $t = 4{,}020$, $u = 2{,}195$; and the working strength for bending

by formula I $\qquad a = 2{,}195\left(1 + \frac{5}{6}\,\frac{\text{min. } B.}{\text{max. } B.}\right)$

Calculation must be made for the most unfavorable strain. For the same iron, under the ordinary strain, $u = 2,195$, and $t = 3,290$. This shows that such axle-iron is a metal which can hardly be suited for bridge-building. If no greater value is given, we put

$$\frac{t - u}{u} = \frac{3,290 - 2,195}{2,195} = \tfrac{1}{2},$$

and $$a = 2,100\left(1 + \tfrac{1}{2}\frac{\text{min. } B.}{\text{max. } B.}\right)$$

Taking $\frac{1}{3}$ as safety co-efficient, the permissible strain per sq. metre.

$$b = 700\left(1 + \frac{1}{2}\frac{\text{min. } B.}{\text{max. } B.}\right) \qquad (11)$$

Alternating Strain.

For Phœnix iron, Wöhler found $u = 2,190$, $s = 1,170$; hence

$$\frac{u - s}{S} = \tfrac{7}{15} \text{ and}$$

by formula II., if $\frac{1}{3}$ be the co-efficient of safety, we find in round numbers

$$b = 700\left(1 - \frac{1}{2}\frac{\text{max. } B'.}{\text{max. } B.}\right) \qquad (12)$$

Here max. $B >$ max. B'; both values numerical, without sign.

Special Cases.

For pieces continually under dead-load we find from (11), since min. $B =$ max. B,

$$b = 1,050 \text{ kil.}$$

For pieces always strained in one direction, then restored to strainless condition, since $B=0$,

$$b = 700 \text{ kil.}$$

For bridge and roof girders, if p is the weight of structure, and q the total load, per running meter,

$$b = 700\left(1 + \tfrac{1}{2}\frac{p}{q}\right)$$

For parts in which maximum tension and compression are equal, $b = 350$ kil., by (12).

B. Steel.

Tension or Compression only.

For Krupp's cast steel, Wöhler found $t = 7{,}340$, $u = 3{,}510$. Reducing the value of u somewhat, because the differences in strength of steel are considerable, and introducing the safety factor $\frac{1}{3}$;

$$\text{since } \frac{t-u}{u} = \frac{7}{6}$$

$$b = 1{,}100\left(1 + \frac{9}{11}\,\frac{\text{min. } B}{\text{max. } B}\right) \qquad (14)$$

This gives 3-fold security if $t = 6{,}000$ and $u = 3{,}390$. This value of t by formula (9) answers to a steel of about 0.6 per cent. carbon, which is suited to bridges. Wöhler found u for axle steel of Krupp, Bochum, Seebohm; and Krupp's plate steel between 3,300 and 3,500; for spring-steel, not hardened, of Mayr in Leoben and Krupp, 3,650. Of course the best material should be used for bridges, and it should not contain more than 0.03 per cent of phosphorus.

Alternating Strains.

For the same cast axle-steel, Wöhler found $s = 2{,}050$; if $u = 3{,}510$, say 3,300, and safety factor is $\frac{1}{3}$;

$$\text{since } \frac{u-s}{u} = \frac{5}{12}$$

$$b = 1{,}100\left(1 - \frac{5}{11}\frac{\text{max. } B'}{\text{max. } B}\right) \qquad (15)$$

This gives 3-fold safety for $u = 3{,}300$, below which value the original strength of steel did not fall in Wöhler's experiments, and for $s = 1{,}800$, while in Krupp, Borsig and Bochum axle-steel the vibration-strength was about 2,000.

Special Cases.

For permanent strain under constant load (14) $b = 2{,}000$ kil. For parts always strained in the same direction, then restored, $b = 1{,}100$ (14) and (15). For bridge and roof girders, and generally for pieces for which

$$\text{min. } B \div \text{max. } B = \frac{p}{q}$$

$$b = 1{,}100\left(1 + \frac{9}{11}\frac{p}{q}\right)$$

For parts under equal max. tension and compression $b =$ 600 kil., (15).

C. Remarks.

The safety factors, and permissible strains for steel and iron, have special reference to bridges and large structures. Hitherto the permissible strain for wrought-iron has been set

at 700. But it is found that b, for wrought-iron, may vary between 350 and 1,050. The most favorable case is that of dead load, the most unfavorable that of alternating tension and compression. In this we see how varyable are the figures required for the safety of the different parts of a structure. Hitherto much material has been wasted in building. It is of no avail to the general security of a structure to employ 700 in places where from 700 to 1,050 may be required, and then to employ 700 in a place where 350 is ample. If there is only one diagonal or vertical in a bridge, which suffers nearly equal strains of compression and tension, the security is only half as great as has been assumed up to the time of Wöhler's investigations. It would certainly be wise to strengthen such exceptionally weak points, and so strengthen the entire structure.

The above values of b, for wrought-iron, give 3-fold security if $t = 3{,}150$, $u = 2{,}100$, $s = 1{,}050$. Wöhler puts 1,100 for permanent structures, in case of alternation of strained and strainless conditions, under tension only, or compression only; and 580 for equal tension and compression; the previous figures being 700 and 350. These correspond to a safety factor of $\frac{1}{2}$. For temporary structures, the values of u and s are greater than we have assumed in (3). For the present this will answer, by taking all values as given above, and selecting another safety factor, say $\frac{1}{2}$ under favorable conditions.

We have not derived the value of b from Wöhler's tests of Krupp's spring steel, because the values of u, s, and t are not all determined, and because the steel had properties which can be assumed only in exceptional cases. Softer and more extensible metal will always be used for bridges. If with this, the ultimate resistance diminishes, it does not follow that it does so in the same ratio as the original strength; for this depends also on the ductility. In the case of hardened spring-steel, with diminishing t, $\frac{u}{t} = \frac{1}{2.50}$: for steel not

hardened, $\frac{1}{2.20}$; for cast axle-steel, $\frac{1}{2.08}$; and for iron, $\frac{1}{1.83}$ to $\frac{1}{1.5}$. Hence estimate of working strength depends upon this. For Krupp's spring-steel, Wöhler's bending tests give

$$a = 3,650 \left(1 + \frac{6 \text{ min } B}{5 \text{ max. } B}\right)$$

for the same hardened,

$$a = 4,390 \left(1 + \frac{3 \text{ min. } B}{2 \text{ max. } B}\right)$$

If very low steel is used for a bridge, the permissible strain must be less. For example, if only 0.45 per cent. carbon is desired, and a minimum ultimate strength of about 5,200 kil. is prescribed, (14) and (15) may be changed to

$$b = 1,000 \left(1 + \frac{3 \text{ min. } B}{4 \text{ max. } B}\right) \qquad (14\,a)$$

$$b = 1,000 \left(1 - \frac{1 \text{ max. } B'}{2 \text{ max. } B}\right) \qquad (15\ a)$$

These formulas, for $t = 5,200$, $u = 3,000$, $s = 1,500$ give three-fold security.

For the arch-bridge at the Champ de Mars, of Bessemer steel, the permissible stress for all parts, whether under tension or compression, was put at 1,000 kil. There are smaller cast-steel bridges in Holland and one of puddled-steel in Sweden. The most important is the bridge at St. Louis, over the Mississippi, which has a middle span of 158.5 m., and two end spans of 152.4 m. The advantages of steel are its greater security against intense cold and its lightness. The difference in ex-

pense will not long stand in the way, as the cost of steel is diminishing.

§ 14.

Calculation of Dimensions.

The formula $F = \dfrac{\text{max. } B}{b}$ is employed in the new calculation of dimensions which differs only in the choice of b. In the following reference is to tension and compression only.

A. Trusses.

For wrought iron; (a) stress in one direction; (b) alternating compression and tension—

$$(a) \quad b = 700\left(1 + \frac{1}{2}\,\frac{\text{min. } B}{\text{max. } B}\right) \qquad (11)$$

$$(b) \quad b = 700\left(1 - \frac{1}{2}\,\frac{\text{max. } B'}{\text{max. } B}\right) \qquad (12)$$

In the second formula max. B is the greater and max. B' the less of both max. strains of different signs. The numerical values are substituted without sign. For girders with uniform load,

$$b = 700\left(1 + \frac{1}{2}\,\frac{p}{q}\right) \qquad (13)$$

p being the weight of girder and q the total load per unit of length. For steel employ (13) (14) (15).

Example.—To find permissible strains and sections for all parts of the half-truss, Fig. 6. Calculations by Ritter's method, which is especially fitted in application of the new estimate of dimensions, because both limiting values of stress are easily

found from one equation, (Calculations for rivets in § 19). Weight of structure 1,000; total load 6,000 kil. at each support. For all sections of girder

$$b=700\left(1+\frac{1}{2}\cdot\frac{1}{6},\right)=758 \text{ kil.}$$

$$F=\frac{\max. B}{758}$$

For vertical VI.

$$b=700\left(1+\frac{1}{2}\cdot\frac{1{,}875}{15{,}625}\right)=742 \qquad (11)$$

$$F=\frac{15{,}625}{742}=21.1, \text{ sq. cm.}$$

For diagonal IX.

$$b=700\left(1-\frac{1}{2}\,\frac{4{,}600}{9{,}550}\right)=531.$$

$$F=\frac{9{,}550}{531}=18, \text{ sq. cm.}$$

For all parts we find

	I	II	III	IV	V	VI	VII	VIII	IX	X
$b=$	758	758	758	758	742	742	688	688	531	531
$F=\frac{\max. B.}{758}$		31.7	39.2	27.7	29.8	21.1	22.4	15.8	18.0	12.7
Old value of F. $\frac{\max. B.}{700}$		32.9	40.7	28.8	30.3	21.4	21.1	14.9	13.1	9.2

The differences in larger bridges are much greater. See next example and § 30.

B. Simple Plate-Beams.

Let M_x be the greatest moment for any section x, and a the distance of the extreme fibre from the neutral axis; then the moment of inertia of the useful section is

$$I = \frac{\max. M_x. a}{b}.$$

The section F of the girder is found by the approximate formula

$$F = \frac{\max. M_x.}{b h_0} - \frac{1}{6} d h_0$$

in which h_0 is the the distance of the centre of gravity of the girder, and d the thickness of the vertical plate. If this is cut by rivets, $\frac{3}{4} h$ is usually taken.

If the calculation is based upon a uniform structure of weight, and a uniform load q, then all along the girder

$$b = 700 \left(1 + \tfrac{1}{2} \frac{p}{q}.\right) \qquad (13)$$

But if the calculation is for concentrated loads, a curve for max. M_x is found, and one for min. M_x for weight of bridge only; then at any section x,

$$b = 700 \left(1 + \tfrac{1}{2} \frac{\min. M_x}{\max. M_x}\right)$$

If the girder is of constant section, so that only the max. moment M_x for concentrated load is determiend,

$$b = 700 \left(1 + \tfrac{1}{2} \frac{\min. M}{\max. M}\right)$$

But a constant section is usually employed only for very

small beams; and in this case the weight of beam may be neglected, and we may assume $b = 700$.

For rail-stringers, because of effect of shock, b should be less, say 650.

Since b is assumed as constant for simple beams, the calculations can be made graphically. For variable b the graphic process is the same as in continuous girders.

Example.—For bridges of dimensions stated below, Laissle and Schübler give the assigned values of $\frac{p}{q}$. To find the permissible strain b.

$$\text{From (13)};\ b = 700\left(1 + \frac{1}{2}\cdot\frac{1}{3}\right) = 817 \text{ kil., etc.}$$

Hence for

$l =$	7	10	15	20	30	40	60	100 m.
$\frac{p}{q} =$	$\frac{1}{8.3}$	$\frac{1}{6.2}$	$\frac{1}{4.7}$	$\frac{1}{4.2}$	$\frac{1}{3.5}$	$\frac{1}{3.0}$	$\frac{1}{2.4}$	$\frac{1}{1.9}$
$b =$	742	757	774	783	800	817	846	884 kil.

For large bridges the permissible strain for equal security is considerably increased. Large and heavy bridges, of course, are less affected by passing load than those of lighter construction.

C. Continuous Girders.

The usual practice is to construct the graphic curves of positive and negative maximum moments. For both; with equal section x (Fig. 7), for wrought-iron

$$b = 700\left(1 + \frac{1}{2}\,\frac{\text{min. } M'}{\text{max. } M}\right).$$

$$b = 700 \left(\frac{1 \text{ max. } M'_x}{2 \text{ max. } M_x} \right)$$

the first, if moments are of same sign, the second, if of different sign.

To determine the sections of continuous girders the method is the same as in previous cases. But the formula

$$F = \frac{\text{max. } M_x}{b\, h_o}$$

is usually employed, the web not being regarded.

The graphic method is mostly used in calculations of sections of flanges of continuous girders, by means of the curve of absolute max. moment, which is formed of the curves of negative and positive moments by revolving the latter about the axis of abscissas (Fig. 7). As the distance h_o of the centre of gravity is assumed as constant, the curve of maximum M_x to another scale gives directly the max. strains of the flanges

$$\text{max.} = B \frac{1}{h_o} \text{max. } M_x$$

and for constant value of b,

$$F = \frac{\text{max. } B}{b}$$

as by the ordinary method.

Further calculation is similar; the only difference being that instead of the curve of max. M_x, a reduced curve is employed. Since

$$F = \frac{\text{max. } B}{b} = \frac{\text{max. } B}{700} \frac{700}{b},$$

it follows that in order to obtain graphically the sectional area we employ a curve corresponding to the equation

$$\text{red. max. } M_x = \frac{720}{b} \cdot \text{max. } M_x \,. \qquad (17)$$

Example.—Given a continuous girder with spans 52, 65, 65, 62 metres. Taking moments, etc., gave for the second span the curves shown in Fig. 7. To find the curve of reduced max M_x for this span.

$$\text{For } x=o \begin{cases} b = 750\left(1+\frac{1}{2}\cdot\frac{482}{2{,}587}\right)=765 \text{ kil.} \\ \text{red. max. } M_x = \frac{700}{765}\cdot 2{,}587 = 2{,}367 \text{ m. kil.} \end{cases}$$

$$\text{For } x=33.1 \begin{cases} b = 700\left(1-\frac{1}{2}\cdot\frac{230}{1{,}823}\right)=656 \text{ kil.} \\ \text{red. max. } M_x = \frac{700}{656}\cdot 1{,}823 = 1{,}945 \text{ m. kil.} \end{cases}$$

$$\text{For } x=51.2 \begin{cases} b = 700\left(1-\frac{1}{2}\cdot\frac{721}{728}\right)=354 \text{ kil.} \\ \text{red. max. } M_x = \frac{700}{354}\cdot 728 = 1{,}440 \text{ m. kil.} \end{cases}$$

The values in the following table were found for given values of b and red. max. M_x; giving the curve of the latter shown in Fig. 7. The curve of max. M_x by which the sections were formerly determined is shown, for the sake of comparison.

b_x	Max M_x	Max M'_x	b	Red. Max M_x
0	−2,587	−482	765	2,367
4	−1,650	−330	770	1,500
8.5	− 970	− 0	700	970
14.1	+ 611	−502	412	1,038
23	+1,470	−300	629	1,636
33.1	+1,823	−230	656	1,945
42	+1,560	−400	611	1,787
51.1	− 728	+721	354	1,440
57	−1,275	+179	651	1,371
59	−1,570	0	700	1,570
62	−2,050	−220	737	1,947
65	−2,776	−390	751	2,587

NOTE.—The values $p = 2{,}000$, $q = 2{,}200 + 4{,}500$, were assumed in the above calculation. Suppose the opening of 65 m is to be spanned by a simple girder, and that $p = 2{,}700$, and $q = 2{,}700 + 4{,}500 = 7{,}200$; then, for the curve of max. M_x, that is of total load, we have

$$\max. M_x = \tfrac{1}{2}\, q\, x\, (l - x) \qquad 3.6\, x\, (65 - x).$$

For the flanges

$$b = 700\left(1 + \tfrac{1}{2}\cdot\frac{2.7}{7.2}\right) = 851 \text{ kil.} \qquad \text{(constant)}$$

In Fig. 7 this curve is reduced to one in which $b = 700$ kil. The ratio of the areas below the hatched and the dotted line shows the approximate ratio of material required for continuous and simple girders. The saving in consequence of continuity is less by the new than by the old calculations; on the other hand, the objection that certain pieces are alternately under tension and compression holds no longer, since we know how to provide for this.

SHEARING RESISTANCE. RIVETING.

That the stability and duration of iron structures depend upon carefully arranged connections needs no proof, yet the calculation and distribution of riveting is generally according to pattern. That harm has not oftener come from this is due to the fact that, after hot-riveting, the contraction of the rivet shank caused by cooling produces a great amount of friction between the riveted parts.

This friction generally amounts to from 800 to 1,600 kil. per sq. cm. of rivet section, and depends on the length of the shank, and at first alone takes up the greater part of the stress on the joint. But, at the moment of rupture, before which there is a slipping of the plates and a deformation of the rivet-heads, friction is not to be depended on; besides, it may disappear as a consequence of shocks, so that all the stress comes on the rivets. Hence friction is left out of account, and dependence is put on shearing resistance alone.

What is to be understood by shearing resistance? Hitherto the name has been given to that shearing strain along a unit surface, which is just sufficient to cause separation. But this is only the resistance to a single dead-load; while, according to Wöhler's law, less strains, repeatedly applied, may produce separation. One feels that this must be true. If we attempt to break the connection in Fig. 8, by shearing off the bolt with the hand, we first try a single pull P; if this is not enough, we pull again and again, and possibly succeed by repeated efforts. Hence the greatest shearing resistance is against dead-load, is at the mean for alternation of strain and strainless condition, and is least when there are alternat-

ings trains in opposite directions; and in this case also the friction is most easily overcome.

Wöhler's law and results have not been applied to the case of shearing strength. But it avails nothing to calculate for a diagonal of a bridge under variable stress if no other stress than that of dead-load is supposed to act upon the rivets. If the rivets break, it is a matter of indifference whether the diagonal falls into the water in one or two pieces. Security and economy can be had only by application of Wöhler's law.

§ 15.

Ultimate Shearing Resistance.

It is usual to give only the ratio of shearing resistance t' to the tensile t (the latter in the direction of rolling). The value of $\frac{t'}{t}$ is put at $\frac{4}{5}$ or 1 by most authorities. But it is admitted that 1 is somewhat too large. For perfectly isotropic bodies, the theory of elasticity gives $\frac{t'}{t} = \frac{1}{1+n}$ and the ratio of the elastic moduli $\frac{E'}{E} = \frac{1}{2(1+n)}$.

Navier, Poisson, and Clapeyron deduced, theoretically, a value $n = \frac{1}{4}$ for all isotropic bodies; hence

$$\frac{t'}{t} = \frac{4}{5}; \quad \frac{E'}{E} = \frac{2}{5}.$$

Later researches, by Cauchy, Lamé and Kirchoff show that theory may give a value of n, between o and $\frac{1}{2}$, so that $\frac{t'}{t}$ lies between 1 and $\frac{2}{3}$, and $\frac{E'}{E}$ between $\frac{1}{2}$ and $\frac{1}{3}$. The values

must be settled by experiment. The experiments of Kirchoff, Wertheim, Regnault and others, made to determine n, are of less value in respect to ordinary dimensions than for those examined. They gave n between $\frac{1}{4}$ and $\frac{1}{3}$ for steel and iron, so that $\frac{t'}{t}$ may be taken between $\frac{4}{5}$ and $\frac{3}{4}$, $\frac{E'}{E}$ between $\frac{2}{5}$ and $\frac{3}{8}$.

Wöhler, under certain hypothesis, finds $\frac{t'}{t} = \frac{4}{5}$ and $\frac{E'}{E} = \frac{2}{5}$ when the shearing edges do not lie in the same plane (Fig. 1). With tests of bars torn and sheared by dead-load $\frac{t'}{t} = \frac{350}{445}$ for Phœnix axle-iron; $\frac{550}{738}$ for Krupp's cast-steel plate: $\frac{4}{5}$ exactly in first case; nearly that in the second. For torsion $\frac{t'}{t}$ was only a little less, and $\frac{E'}{E} = \frac{1.95}{5}$. Bauschinger obtained the same value exactly in a series of experiments on Bessemer steel (See § 11). The common theory that torsion is a kind of shearing, Spangenberg concludes to be true, because of the appearance of fracture surfaces; and until further investigation is made the same figures may be taken for torsion as for shearing. The experiments also confirm the relation between tension and shearing which is deduced from the theory of elasticity. That caution must be used in applying these results to anisotropic material is clearly shown by late experiments by Bauschinger. Resistance to shearing was of very different values, depending on the angle between its plane and

that of the direction of rolling. Pauschinger gives six different dispositions (Fig. 9). Of these I, III, IV, are of practical import. In I, III, IV, t' was but little smaller than t for tension in the direction of rolling; in II the difference was considerably greater; in V and VI the average was $t' = \frac{1}{2} t$, and the value sank to $\frac{1}{3}$. The differences were greater in proportion to the degree of fibrous and laminated condition.

Rolled-iron from Wasseralfingen, for an initial value $t =$ 3,893, gave t', I 3,448, II 2,836, III 3,590, IV 3,060, V 1,787, VI 1,767. For two kinds of Lothringer angle-iron, for $t =$ 3,160, $t' =$ I 2,630, III 3,030, IV 2,620; or $\frac{t'}{t}$ greater than $\frac{4}{5}$. For several German, French and English boiler plates, for $t =$ 3,180, these were the results; I 2,410, III 2,460, IV 2,540; mean about $\frac{4}{5}$ of 3,180. For Styrian cast-steel plate for locomotive boilers t' I 3,920, III 4,380, IV 4,460; for $t = 5{,}025$; *i. e.*, $t' : t$ greater than $\frac{4}{5}$.

It may be said in general, that in the practical cases I, III, IV, $\frac{t'}{t}$ $\frac{4}{5}$ is the best ratio.

It may happen, especially with boiler-plate, that for III and IV, $t = t'$ or $t < t'$; but this must not be taken for granted.

§ 16.

Permissible Shearing Stress.

It follows, from Wöhler's tests, in which the shearing edges lay in different planes (Fig. 1), and Bauschinger's, in which they lay close, that the ratio $\frac{t'}{t}$ depends on this

difference in some way not understood. Wöhler found that the shearing and torsion strength for repeated alternating strains could be assumed as $\frac{4}{5}$ths of the ultimate tensile strength in the direction of rolling, under like conditions (*i. e.*, equal ratios of limiting stresses), as was to be inferred from theoretic considerations. This relation, then, may be assumed as holding in practical cases, (I, III, IV). Then t', u', s' and a', are found by multiplying each corresponding value by $\frac{4}{5}$.

For stress in one direction

$$a' = u'\left(1 - \frac{t'-u'}{u'}\right)\frac{\text{min. } B}{\text{ax } B}$$

$$= \frac{4}{5}u\left(1 + \frac{t-u}{u}\cdot\frac{\text{min. } B}{\text{max. } B}\right) = \frac{4}{5}a \qquad \text{(III)}$$

This applies to the riveting of those members which are always under tension or always compressed; for the sections of plate girders near supports, and for riveting of boilers.

For stresses in opposite directions

$$a' = u'\left(1 - \frac{u'-s'}{u'}\right)\frac{\text{max. } B'}{\text{max. } B} \qquad \text{(IV)}$$

$$= \frac{4}{5}u\left(1 - \frac{u-s}{u}\,\frac{\text{max. } B'}{\text{max. } B}\right) = \frac{4}{5}a.$$

This applies wherever there is alternate pull and thrust.

A. Wrought-Iron.

For strain in one direction

$$b' = \frac{4}{5}b = 560\left(1 + \frac{1}{2}\,\frac{\text{min. } B}{\text{max. } B}\right) \qquad (18)$$

and for opposing stresses

$$b' = \frac{4}{5} b = 560 \left(1 - \frac{1}{2} \frac{\text{max. } B'}{\text{max. } B}\right) \qquad (19)$$

For torsions the ratios are to be got from the limiting torsion moments; then (18) and (19) give the permissible strain in the outermost fibers. For dead-load, $b' = 840$; for cases of stress and restoration alternating, $b' = 560$; for equal opposing stresses, $b' = 280$.

B. Steel.

For one direction

$$b' = \frac{4}{5} b = 880 \left(1 + \frac{9}{11} \frac{\text{min. } B}{\text{max. } B}\right)$$

For opposing stresses

$$b' = \frac{4}{5} b = 880 \left(1 - \frac{5}{11} \frac{\text{max. } B'}{\text{max. } B}\right)$$

For dead-load $b' = 1{,}600$; for alternate stress and restoration $b' = 880$; for equal opposite stresses $b' = 480$.

C. Remarks.

Possibly in cases of shearing and torsion acting in opposite directions the above may seem to be anomalous; but Wöhler's experiments confirm the results. For Krupp's cast axle-steel $u = 3{,}510$, $s = 2{,}050$, and $u' = 2{,}780$, s' 1,610; and by direct and indirect process,

$$a' = u' \left(1 - \frac{u' - s'}{u'} \frac{\text{max. } B'}{\text{max. } B}\right)$$

$$= 2{,}780 \left(1 - 0.42 \frac{\text{max. } B'}{\text{max. } B}\right)$$

$$a' = \frac{4}{5} u \left(1 - \frac{u-s}{u} \frac{\text{max. } B'}{\text{max. } B} \right)$$

$$= 2{,}808 \left(1 - 0.42 \frac{\text{max. } B'}{\text{max. } B} \right)$$

If steel, as low as that supposed in (14a) and (15a), then

$$b' = \frac{4}{5} b = 800 \left(1 + \frac{3}{4} \frac{\text{min. } B}{\text{max. } B} \right) \qquad (20a)$$

$$b' = \frac{4}{5} b = 800 \left(1 - \frac{1}{2} \frac{\text{max. } B'}{\text{max. } B} \right) \qquad (21a)$$

If the stress for II, Fig. 9, is to be found, we may put $b' = \frac{3}{4} b$; while for V and VI, which are not of practical import, b' should not be more than $\frac{1}{2} b$.

§ 17.

Web of Plate-Girders.

The ordinary calculations do not give sufficient values for the thickness of the vertical web of plate-girders (Fig. 41). Experiments have proved that the plate is most easily destroyed by lateral buckling; but the forces involved cannot be analyzed. To prevent this the girder is stiffened, and the web should not be too thin. It is to be considered that the plate may be weakened by corrosion, and that the pressure on the rivets may be too great. The effects of other strains must be considered. The horizontal tension and compression are proportional to the distance from the axis of gravity, and is therefore less at any other place than at the outermost fibre, and less than the permissible value. The

horizontal shearing resistance for vertical unit across the breadth, called *specific*, is greatest at the axis, viz.:

$$H_o = \frac{\text{max. } V_x}{h};$$

h being the distance from the centre of tension and compression. In order to resist the horizontal shearing at any distance from the axis, a plate is required of thickness d, found by the formula,

$$1\text{ d.}\frac{4}{5}b = \frac{\text{max. } V}{h}$$

$$d = \frac{5}{4}\,\frac{\text{max. } V_x}{hb}$$

This will also answer for vertical shearing; the vertical and horizontal stresses being equal at all points: making $h = \frac{9}{10}$ths h_o; h_o being the distance between centres of flanges; we have

$$d = \tfrac{25}{18}\,\frac{\text{max. } V_x}{bh_o}.$$

Oblique strains may be greater than the vertical and horizontal, if the vertical shearing V_x and the moment M_x have high values at the same time, as at the posts of continuous bridges and in small girders under concentrated loads. But only the oblique tensions and compressions are of higher value, the highest being at the junction with the flange. In unfavorable cases in which regard is had to oblique stress, take $\frac{10}{9}$ths of max. V_x for simple girders, $\frac{5}{4}$ths for continuous;

$$d = \frac{3}{2}\cdot\frac{\text{max. } V_x}{bh} \qquad (22)$$

$$d = \frac{5}{3} \cdot \frac{\max. V_x}{b h_o} \qquad (23)$$

These are the formulas of Laissle and Schübler.

The values of b for wrought iron in the respective cases when the shearing is always in the same direction, as near supports; and when it acts in opposite directions, as near the middle of spans, are—

$$b = 700 \left(1 + \frac{1}{2} \; \frac{\min. V_x}{\max. V_x}\right)$$

$$b = 700 \left(1 - \frac{1}{2} \; \frac{\max. V'_x}{\max. V_x}\right)$$

Formulas (22) and (23) give the maximum value only of $\frac{\max. V_x}{b}$; this always occurs at a support, so that only the first formula for b is employed.

For a simple plate-girder with uniformly distributed load, max. $V_x = \frac{1}{2} q\, l$; min. $V_x = \frac{1}{2} p\, l$; hence

$$b = 700 \left(1 + \frac{1}{2} \frac{p}{q}\right)$$

and by substitution in (22)

$$d = \frac{q\, l}{930 + \quad 1 + \frac{p}{2\, q} \Big) h_o,}$$

in which p = weight of girder, q = total load in met. kil. per meter; l = width of span in metres; h = distance between centres of gravity in centimeters.

Example.—For simple plate girders of l = 7 and 10 m.

of span ; h_o 70 and 100 ctr. ; $p = 900$, $q = 8{,}100$; $p =$ 1,100, $q = 6{,}600$ kil.

What thickness of web answers for all forces ?

For $l = 7$
$$d = \frac{8{,}100.7}{930\left(1 + \frac{1}{18}\right)70} = 0.82 \text{ cm.}$$

For $l = 10$
$$d = \frac{6{,}600.10}{930\left(1 + \frac{1}{12}\right)100} = 0.66 \text{ cm.}$$

For long girders d may be increased to 0.9 or 1.

§ 18.

Method of Riveting.

Whether rivet holes should be drilled or punched is a question upon which opinion is divided. Two points are to be considered : (1) the probability of good riveting ; (2) the resistance of good riveting. For the first, drilling is the better.

The strength of riveting depends on the resistance of the plate and on the strength of the rivets. The strain upon the middle portion of Fig. 10 must be transmitted by the fibres at the circumference of the rivet hole ; these must, therefore, receive more than the average stress.

The chief advocate of punching is Fairbairn ; his main argument, that it tests the quality of the metal. He repeatedly says that bad iron will tear with punching, and would be rejected, and that this is decisive. It is true that the worst iron would tear visibly ; but it does not follow that other kinds would not be injured. The following table of the average of

latest results of experiments in America shows how selected iron is weakened by punching :

Test-bar 44 mm. broad, 8 mm. thick. Diam. of rivet-hole, 16 mm.	Result.	Load for sq. cm. of trans. section.	Load for sq. cm. of rivet section.
1. Plate entire............	Plate torn.	4,200	
2. " drilled..........	"	3,530	
3. " punched.........	"	2,690	
4. Single-shear riveting plate drilled...............	Rivet sheared.	3,280	3,750
5. Single-shear plate, punched..................	Plate torn.	3,510	4,030

Hence the strength of the punched plate was 0.64, that of the drilled, 0.84 of the strength of entire plate. Sharp, in his tests, found that, with Bessemer plate, the strength of punched was only 0.59 that of drilled, the per cent. diminishing with increased brittleness, thickness of plate, and with the decrease of the ratio $\frac{e}{d}$ of the breadth of the bar to the diameter of the hole.

The like does not hold true with regard to the tenacity of the rivets. In results (3) and (5) the friction per sq. ctr. of rivet-section was $3,510 - 2,690 = 820$ k., so that in case (5) the rivet bore a strain of $4,030 - 820 = 3,210$ k. In case (4), though friction was equal in both cases, it was sheared by $3,750 - 820 = 2,930$ k. Hence the rivet in the drilled hole bore about 9 per cent. less strain than that in the punched. Still it is obvious that the strength of the rivet cannot depend upon the form of the hole.

Notwithstanding all objections, the difference can be ascribed only to the sharper edges of drilled holes, which makes them cut more easily. This explains the fact that in Sharp's and

Kirkaldy's experiments the same sized rivets bore a much less strain in hard steel plate than in iron plate. Fairbairn observed that by rounding off the edges of the rivet-holes the rivet-strength, with drilled holes, was increased about 12 per cent.; but, with punched holes, the increase was only $2\frac{3}{4}$ per cent. Assuming that in experiment (5) above, 3,210 is the strength of the rivet in the punched hole, not rounded (this is about $\frac{4}{5}$ths of the tensile strength of very good iron plate, so that the rivet, at the time of the rupture of the plate, was strained to near the limit of resistance), we find that by rounding off the holes we get: for punched holes, $3{,}210 \times 1.0275 = 3{,}298$; for drilled holes, $2{,}930 \times 1.12 = 3{,}281$ k. So exact an agreement is not always to be expected, since the effect of the rounding depends on the brittleness and thickness of the plate, &c.; but the difference in the hold of rivets in the two cases is explained as above. We conclude that while the resistance of plate is greater with drilled than with punched holes, the hold of the rivets in the first is not less than in the second case, if the edges are rounded. This treatment of the edges and countersinking of the rivet-heads have the advantage of increasing the surface to be sheared.

Fairbairn's results in other respects do not disprove this view. Effective shearing strains were as follows:

Hole.	Load per sq. cm.	Riveting.	
Punched	3,080	By machine	Single shear.
Drilled	2,920	"	
Punched	3,240	hand	
Drilled	3,200	hand	
Drilled and rounded	3,190	machine	
Punched	6,970	hand	Double shear.
Drilled	6,170	"	
Drilled and rounded	7,250	"	

Fairbairn's inference of greater strength for punched holes was due to the fact that he did not take into account the decided difference in the diminution of resistance of drilled and punched plates; this not regarded, judgment must be in favor of punching.

In general, then, drilling is to be preferred to punching; but there may be modifying circumstances which make punching preferable. In this case the punched parts must be at least "hand-warm". All parts that are at all torn must be rejected; bad-fitting holes should be carefully reamed down; the parts near the holes must be heated, this being important when the metal is brittle, and indispensable for steel. Sharp found that the strength of punched steel-plate, 0.8 cm. thick was raised from 3,300 to 5,200 kil.; that of drilled plate without heating, was 5,600. The difference 400 may have been loss of ultimate strength caused by heating. Kirkaldy found that by hardening, the strength of punched plates was increased more than by heating; but it must be remembered that this diminishes the resistance to impact.

Seven experiments by Fairbairn gave a mean of $7\frac{1}{4}$ per cent. greater strength for hand than for machine riveting. This Fairbairn attributes to the fact that when riveting is done by hand the rivet is hardened by being hammered while cool. He favors machine riveting in other respects, because the holes are more surely plugged, and the rivets hardly ever work loose. Hot riveting is better than cold, because a greater frictional resistance ensues, and because the closer joints keep out corrosion. If the total thickness of plate is more than 10 cm., it is better not to employ hot riveting, because the contraction might shear a rivet or spring up its head. For shank-lengths of more than 15 cm., heat should not be used. But in this case cold riveting will hardly answer, and turned screw-bolts of slight taper, say $\frac{1}{100}$, may be used. Gerber says that such bolts have from 6 to 8 per cent. more resistance than hot-riveting, because of the more complete filling of the holes.

Aside from questions about machine or hand riveting and cold or hot riveting, which circumstances must decide; the following is the result of our investigation. *Best riveting:* holes drilled; edges partly rounded; rivets countersunk. *Good riveting:* holes punched hand-warm; bad fitting ones broached down; edges rounded; heads countersunk; heated near holes. The latter is superfluous for very thin plates only.

§ 19.

Elastic Relations.

Riveting is single, double, &c., according to the number of rows of rivets in the direction of strain. If a rod S (Fig. 11) is riveted to a non-elastic body K, by several rows, the row I must still bear the entire strain B, for the part of B assigned to II must act by tension on $I\ II$, tending to stretch it. This cannot happen, because I does not yield on account of the deficient elasticity of K, so the part of B assigned to II is transferred back to I by compression. It follows that if an elastic body is fastened to one that is non-elastic, multiple riveting is inefficient, since the outermost row must bear the whole strain.

If K were elastic, but less so than S, being of equal or greater cross-section, a part of B would act upon II, but only so much as is required by the possible elongation of $I\ II$. I must bear all the residue. Hence multiple riveting of bodies of different elasticities, *e. g.*, steel and iron, or cast and wrought-iron, is not to the purpose.

But like relations hold for bodies of equal elastic modulus. In Fig. 12, if the diagonal represents the rod S, and the flange-plate the body K; it is plain that the latter must give in the direction of B less than the former, and the riveting I will have to bear more than one-fourth of B, and each of the

row II more than III. If such connections are unavoidable the riveting should be a little more yielding than usual.

If two bodies whose elongations for the same stress are nearly equal, are riveted double, or triple, they strive to attain unequal elongations between rivets, because the forces acting on the two adjacent parts are not equal. Denote the stresses on the rivets (Fig. 13) and the surfaces intervening, as in the diagram; then

$$I = I'$$

$$I\ II \quad I'\ II' = B - I$$

$$II = II'$$

$$III = III'$$

$$II\ III = II'\ III' = B - I - II.$$

The parts $I\ II$ and $II'\ III'$ are therefore under the action of forces of different magnitudes, viz.; $B - I$ and $B - I - II$. The rivet I cannot yield to the elongation of $I\ II$, and a portion of this force must act as pressure on I. The same holds true of the portions $II\ III$, $I'\ II'$, and the rivet I'. Hence the weak point of every riveting which is more than double, lies near the outermost rviit in the direction of the strain. In such cases it is better to exceed the number calculated by the ordinary method. In some cases a direct remedy may be employed. The elongations of the pieces on both sides would be equal, and the most favorable distribution of B would be determined, if the sections of the pieces were proportional to the forces acting on them; hence in Fig. 13.

$$\frac{F_1}{F_1'} = \frac{B - I}{B - I - II} = \frac{F_2'}{F_2}$$

This ratio can be obtained when an alteration of section is admissible, by stepped offsets; but forms like Figs. 14, 31, 34, 35 are to be preferred.

If two bars are so riveted that the stress acts outside the gravity-axis of one of them, then in the other, besides the elongating or shortening force, a bending couple acts which causes unequal strains upon the rivets. In all cases the rivets of both should be set as symmetrically as possible.

Fig. 15 represents a splice-plate for which Schwedler found, by Navier's theory, the maximum strain per sq. cr. for the outermost fibre

$$K = b\left(\frac{4}{\alpha} - \frac{3}{\alpha^2}\right)$$

in which $b = \frac{B}{F}$ is the calculated stress per sq. cr., and $\alpha = \frac{e'}{e}$

K would have its maximum at $\alpha = \frac{3}{2}$ and, *e. g.*, for

$\alpha = 1$	$\frac{5}{4}$	$\frac{3}{2}$	3	5
$\frac{K}{b} = 1$	1.28	1.33	1	0.68.

i. e., the resistance of the plate, so far as upon one side it projects by less than double the width of the bar would be increased by cutting away the projecting portion.

Theune has attempted to test this by experiments with plates of caoutchouc, which represented bar and splice-plate cast in one piece. It appeared that in general the weak point did not lie in the neutral axis, but at O.

An inference of disagreement of theory and the cause assigned by Theune for the phenomenon would not be right. In Theune's experiments, the hypothesis of Navier's deflection theory—deflections infinitely small in comparison with the length of the rod—was not verified. The force, B, could not be regarded as acting parallel to the axis of the splice-plate. In Fig. 16, S refers to Schwedler's case, T to that of Theune,

for caoutchouc. To obtain the latter from the former requires a new deflection, in which the outside fibre of the splice-plate is partially freed from strain, and the weak point is moved toward the section O, which is now strained in the same way as the sections over the posts of continuous girders.

We perceive, by referring to the above values for $K : b$, that hardly a perceptible increase of stress upon the splice-plate can come from the projecting portion, and that, cutting away the latter would generally cause a better distribution of strain upon the rivets.

§ 20.

Total Section and Number of Rivets.

A riveting is called single-shear, double-shear, . . . i - shear, if under the strain there occur one, two, three, or i changes of the direction of force (Figs. 17 to 22), for according as the force, B, must shear the rivet in one, two, or more, in order to destroy the junction, the whole, a half, &c., of B operates in shearing off one section. Each i-shear riveting can be regarded as divided into i single-shear, as is shown by the dotted lines in Figs. 19, 20. The plate thickness of each single-shear will hereafter be denoted by δ.

Next we have to determine the total section F_n of the rivets required by a connection of rods or entire plates. Let max. B be the greatest shearing force upon this system; then we have, if b denotes the unit stress for tension and compression, and b' the permissible shearing stress per sq. ctr.

for single-shear $$F_n = \frac{\text{max. } B}{b'} = \frac{\text{max. } B}{\frac{4}{5}\, b} = \frac{4}{5} F \qquad (24)$$

for i-sshar $$F_n = \frac{\text{max. } B}{i\, b'} = \frac{\text{max. } B}{\frac{4}{5}\, i\, b} = \frac{5}{4\, i} \,.\, F \qquad (25)$$

in which F is the total useful section of the rod.

The necessary number n of rivets is easily found. If

$d =$ diam. $\qquad F_n = n \, . \frac{1}{4} \pi \, d^2.$

hence for single shear,

$$n_1 = \frac{d}{\pi \, d_2} . \frac{\text{max.}\, B}{b'} = \frac{5}{\pi \, d_2} . \frac{\text{max.}\, B}{b} \frac{5}{\pi \, \mathrm{d}_2} F \qquad (26)$$

for i-shear

$$n_i = \frac{4}{i \pi \, d^2} . \frac{\text{max.}\, B}{b'} = \frac{5}{i \pi \, d^2} . \frac{\text{max.}\, B}{b} = \frac{5}{i \pi d^2} = F \frac{n_1}{i} \qquad (27)$$

The values of b or b' are not necessary in this calculation if F is correctly determined.

That double-shear riveting is twice as effective as single may be seen in Fairbairn's results, referred to in § 18. The ratio in that case is still more favorable for double-shear riveting. This is because the cutting effect of edges is more certain and effective in single-shear riveting. Straight-shear rivetings have this advantage: they do not produce a couple tending to bend the plate and spring the rivet-heads. When it is difficult to set a sufficient number of single-shear rivets, a forked arrangement, like that in Fig. 24, may be employed, making the riveting double-shear, and of half the number.

The larger the rivets the fewer required, and the further apart they may be set. But it is to be observed that with the strain upon a rivet the corresponding strain upon the wall of the rivet hole increases so that a destruction of the rivet hole may ensue. Although in many cases a little burring is not objectionable, yet we must infer from Gerber's experiments that the rivet-wall should not be strained more than twice the permissible tensile strain per square cr. of its projection, hence

$$\frac{\pi d^2}{4} . \frac{4}{5} \, b = \text{or} < \, d\delta \, . \, 2b \text{ or } d < \text{or} = 3 \, . \, 2\delta$$

remains. This holds both for a single and an i-shear and riveting, and, in the latter case, δ denotes the thickness of plate of each resolved riveting.

The value of d generally lies between $1\frac{1}{2}d$ and $3d$. A fixed ratio $d \div \delta$ as prescribed for general use will not answer in bridge construction where the use of more than two or three sizes of rivets is not convenient.

Remarks.

It has been shown that the strain on a rivet is compounded of a shear across and a tension along due to cooling: and that no certain method of calculating the values is known. But if the strain lengthwise preponderates, then the friction-resistance of from 800 to 1,600 k. will not permit a shear: and if the friction is removed by any cause there is no further longitudinal stress. More unfavorable stresses are not to be expected in the intermediate conditions, since with the ordinary shearing values the tension lengthwise has diminshed considerably at the beginning of shearing.

The initial strain lengthwise is not exactly determinable, since we do not know the co-efficient f of friction. Assuming it as $\frac{1}{5}$th, and recollecting that R, the friction, acts on two sides, we get as value of longitudinal strain per square ctr. of rivet section,

$$L = \frac{R}{2f} = 1{,}200 \text{ to } 2{,}400 \text{ kil.}$$

This would be a large value for the best fine-grained iron for dead-load, if its ultimate resistance were increased by passing the elastic limit (§6). As L increases with the length of the shank what has been said about heating in § 18 applies.

We have assumed $F_n = \frac{5}{4} F$ in accordance with all the results of theory and experiment hitherto obtained; any deviation

from this must have sufficient ground. Reasons for making $F_u = \frac{5}{4} F$ are that rivet iron is better than common rolled iron, and that the bars lose some of their strength by drilling or punching. On the other hand the reasons for making $F_n = \frac{5}{4} F$ are the unfavorable effects of oblique strains and the insufficient distribution of strain among the rivets in plates that yield but little, on account of multiple riveting or unsymmetric grouping. These effects cannot be decisively balanced ; hence there is no ground for changing the ratio $F_n = \frac{5}{4} F$.

Example.—The number of rivets necessary for the fastening of diagonals and verticals of the truss (Fig. 6) is to be found. $d = 2.5$ cm. Riveting single-shear.

$$\text{By (26) } n = \frac{5}{\pi d^2} . F = 0.25\ F$$

For the vertical VI, $F = 21.1$,
hence $n = 0.25 \times 21.1 = 6$.

For the other pieces we have

	II	III	IV	V	VI	VII	$VIII$	IX	X
$F =$	31.7	39.2	27.7	29.8	21.1	22.4	15.8	18.0	12.7
$n =$	8	10	7	8	6	6	4	5	4

For the riveting of the vertex-plates with the flange, see the example in § 25.

§ 21.

Indirect Transmission of Force.

It has been assumed that the bars to be riveted are in close contact. Otherwise the relations are entirely different ; but to this case no attention has hitherto been given.

Suppose that bar I is to be riveted to bar III, Fig. 25, so as to transfer the force, B, from I to III. Suppose bar II lies between I and III, for the present supposed not to be

weaker than I. The force, B, can pass directly from I only to II, and n_1 rivets are required at A. But, if II is not to be strained, it must be freed from an equal strain, B, before A, at D, for which n_1 rivets are required. The indirect transmission, with an intervening plate, therefore, requires twice as many rivets as the direct.

That the force, B, can actually pass directly only from I to II is shown by a glance at Fig. 26; for, in order that the strain may be transmitted to a bar, the rivet must be pressed in the direction of the force against the wall of the rivet-hole in the bar. And the same figures shows how II and III is discharged to III, and that, theoretically, the rivets may be limited by the dotted line. If the bar were compressed, the walls of the rivet-holes would act on the opposite side against the rivets.

If there are two intervening plates, the transmission takes place as in Fig. 27, and 3 n_1 rivets are necessary. And, generally, for any single-shear riveting, indirect transmission with m intervening plates, requires $m + 1$ times as many rivets as direct transmission. Hence from (26) for m intervening plates

$$n = (m + 1)\, n_1 = \frac{5\,(m + 1)}{\pi\, d^2} \frac{\max.\, B}{b} = \frac{5\,(m + 1)}{\pi\, d^2} F \qquad (28)$$

in which F is the useful section of the rod, whose strain, max. B, is to be transmitted. In this equation (26) is applied for the value $m = o$.

The principle and formula (28) serve for any one of the single-shear rivetings, into which an i-shear riveting is resolved. In Fig. 28, *e. g.*, to transmit B from I to III, $2\ \frac{1}{2}\ n_1 = 2n_2$ rivets are required. More than two-shear rivetings are not employed for indirect transmission.

It was assumed above that the intervening plates were not weaker than those whose strain they had to transmit. This limitation is not necessary, and our results hold true in gen-

eral. For example, if, in Fig. 29, the intervening plate *II* were weaker than *I*, it would have been necessary to regard the force transmitted as there indicated, if *II* is at least half as strong as *I*. Hence, giving the rivet-number for direct transmission the same index as the force

$$n = 2\,n_{II} + 2\,(n_I - n^{II}) = 2\,n_I$$

exactly as above required.

Filling-plates, sometimes unavoidable (Fig. 30), need only single riveting. Either they move freely with the rivets, without offering resistance, so that transmission from *I* to *II* occurs at *A*; or they are so disposed that the strain passes from the filling plate at *D* to the plate intended to receive it. But it is to be noted, that in the first case the rivets bear a strong bending stress; for which provision can be made by additional rivets. (Applications in §§ 24 — 27).

22.

Riveting of Bars.

The previous formulas serve to determine the number of rivets of a given diameter for any fastening. The arrangement remains to be considered. In § 19 we found that the rivets should be arranged symmetrically with reference to the axis of each rod; that single riveting is best if the bodies are of equal or different elasticity, and yield in different degrees in the direction of the force; that in all cases more than double riveting brings an unequal strain upon the outermost rivets, and that when possible, the rivets in the outer rows should be set as close as possible.

In riveting bars in the webs of trusses, the piece should be weakened by rivet-holes as little as possible. In the distribution of the rivets of a diagonal, as in Fig. 31, a single

hole must count for weakening, and the useful working section becomes $F = b_I, \delta = (b - d)\,\delta$. For though a breadth of row II less by d serves to transmit the force, still the latter is diminished by the amount taken up by I. If the useful breadth b^I, in the outermost row answers for the whole piece, then for II, III, &c., the following values are sufficient:

$$b_{II} = \frac{B - I}{B} \cdot b_I = \frac{II + III + \ldots}{b} b_I$$

$$b_{III} = \frac{B - I - II}{B} b_I = \frac{III + IV + \ldots}{B} b_I$$

in which I, II, &c., signify the strains taken up by the respective rows. If D are wanting in a row, then only $(b - \delta - D)\,\delta$ would be the useful working section. Hence the number in in two successive rows never increase by more than the number required in the outermost row. After that the only condition is

$$d\,\delta\,b < \text{or} = \frac{\pi\,d^2}{4} \cdot \frac{4}{5} b \quad \text{or } \delta < \text{or} = \frac{2}{3} d.$$

If $\delta = \frac{2}{3}d$, then in row I there should be more rivets than the increase in two successive rows But such thick pieces are seldom used; and it must not be forgotten that δ always denotes the thickness for one-shear riveting.

Since for IV, in Fig. 31, less breadth is required than for III, though less weakening is caused by rivet holes, the section may be diminished in another way. In § 19 it was shown that such a diminution is desirable in order for a uniform distribution of strains on the rivets; so that the ends of the bars are often of the forms shown in Figs. 14, 31.

In pieces under compression account need not generally be taken of the diminution of sectional area by riveting, since the transmission of the strain occurs along the whole

breadth (Figs. 32, 33). Hence it is not of so much consequence that as few rivets as possible should be set in row I; but it is well to rebate, say one-half of the rivet holes, as tending to weaken; because exact contact of hole and rivet is not to be expected at all points.

In order to effect a distribution of strain as uniform as possible at the rivetings, the rivets of each row are set opposite the intervals of the adjacent rows, generally opposite the middle (Figs. 31, 40, 42), each rivet then receives the stress of a strip of breadth determined by the equation

$$\frac{\pi d^2}{4} \cdot \frac{4}{5} b = \beta \, \delta \, b$$

$$\therefore \beta = \frac{\pi}{5} \frac{d}{\delta} d \qquad (29)$$

The stress upon each strip must be transferred to the rivet, hence within each group through each cross section there must run as many strips of breadth β as the number of rivets remaining, provided, of course, that no more rivets are set than required theoretically by 26 and 27. Hence, for the purpose of symmetric and direct transfer, a strip may be carried through in two halves each $\frac{1}{2}\,\beta$ wide, up to its rivet. This grouping (Figs. 34, 35, 39, 40) was first employed by Schwedler. We have now to determine what breadth the strip must have behind the rivet. The useful sectional area of the bar can seldom be an exact multiple of β, but is somewhat less, because for each fraction of the calculated number of rivets a rivet is counted, and sometimes extra ones are added.

The permissible minimum distance e between rivets in the direction of the strain, and the minimum distance r of the last row from the edge, depend upon the condition that the same security is required against shearing of the rivets as against the forcing out of the hatched pieces, Fig. 36. For this, calculation is made for the surfaces under shear between the tangents to

the hole.edges which lie adjacent ; because of the diminution of resistance, mentioned in § 18, in the immediate vicinity of the hole, where small cracks are likely to be caused by punching ; hence

$$P = \frac{\pi d^2}{4} . \frac{4}{5} b = 2 (e - d) \ \delta \ \frac{4}{5} \ b$$

$$\text{and } e = \left(1 + \frac{\pi}{8} . \frac{d}{\delta} \right) d \qquad (30)$$

Generally this value must be made larger for practical reasons, *e. g.*, to allow hammering of the head.

The least distance of the outermost row from the edge is $\frac{1}{2}d$ less than e (Fig. 36); hence

$$r = \left(1 + \frac{\pi}{4} . \frac{d}{\delta} \right) \frac{d}{2} \qquad (31)$$

It appears by (30) and (31) that behind each rivet a strip of at least $\frac{\pi}{8} \ \frac{d}{\delta} \ d$ is requred, with reference to the stress upon this rivet; so that the dimensions of the loop surrounding the rivet as represented in Fig. 37 are known.

For many-shear rivetings the value for δ in all the formulas is the thickness of the plate of the single-shear into which the former have been resolved.

Example.—In practical cases, in which δ is arbitrary, the value $\frac{1}{2}$ d is often well adapted. The magnitudes β, e, r, are to be found under this hypothesis. From 29, 30, 31 we find

$$\beta = 1{\cdot}26 \ d, \quad e = 1.79 \ d, \quad r = 1.29 \text{ d}.$$

Hence, in millimetres, for

$d =$	20	21	22	23	24	25	26	27	28	29	30
$\beta =$	25.2	26.5	27.7	29.0	30.2	31.5	32.8	34.0	35.3	36 5	37.8
$e =$	36	38	39	41	43	45	47	48	50	52	54
$r =$	26	27	28	30	31	32	34	35	36	37	39

The values of r and e are generally made larger.

§ 23.

Riveting of Entire Plates.

The grouping of rivets is determined in advance, being uniformly arranged in one or two rows. If the two plates are of like material and the same strength, double riveting is generally preferable; the necessary number of rivets remains the same, and the strain acts uniformly; but the useful section F' is larger and the strength at the seam is less reduced by the rivet-holes. Conditions that make a very tight joint desirable may prevent double riveting.

The necessary number is not usually found by the formulas of § 20; but depends on the rivet-pitch; this depends upon the condition that there should be the same security against shearing the rivet as against tearing the plate. Hence, for single or double riveting (Figs. 38, 40), since the rivets are always single-shear,

$$\frac{\pi\, d^2}{4} \cdot \frac{4}{5}\, b = (D-d)\,\delta\, b$$

$$2\,\frac{\pi\, d^2}{4} \cdot \frac{4}{5}\, b = (D-d)\,\delta\, b$$

and for single riveting $D = \left(1 + \frac{\pi}{5} \cdot \frac{d}{\delta}\right) d$ (32)

for double riveting $D = \left(1 + \frac{2\,\pi\, d}{5\ \ \delta}\right) d$ (33)

Higher then double is not admissible, because of unequal transmission of strain.

$\alpha_{,}=\frac{D-d}{D}$ gives the ratio of useful to entire section, and for

$$\text{single riveting } \alpha \frac{1}{1+\frac{5}{\pi}\frac{\delta}{d}} \qquad (34)$$

$$\text{double} \quad \text{“} \quad \alpha \frac{1}{1+\frac{5}{2\pi}\frac{\delta}{d}}$$

and for the useful section F, if F' denotes the section not weakened, we have

$$F = \alpha F' \qquad (36)$$

For example, by (34) and (35).

for	$\frac{d}{\delta}=$ 1.5	2	2.5	3
for single,	α=0.49	0.56	0.61	0.65
for double,	α=0.65	0.72	0.76	0.79

If the stress acts perpendicularly to the direction of rolling, and is $\frac{9}{10}$ b (§ 5);

$$\text{for single riveting } D=\left(1+\frac{2\pi}{9}\cdot\frac{d}{\delta}\right)d$$

$$\alpha = \frac{1}{1+\frac{9}{2\pi}\cdot\frac{\delta}{d}}$$

$$\text{for double riveting } D=\left(1+\frac{4\pi}{9}\cdot\frac{d}{\delta}\right)d$$

$$\alpha = \frac{1}{1+\dfrac{9}{4\,\pi}\cdot\dfrac{\delta}{d}}$$

and, *e. g.*, if	$\dfrac{d}{\delta} =$	1.5	2	2.5	3
for single,	$\alpha =$	0.51	0.58	0.64	0.68
for double,	$\alpha =$	0.68	0.74	0.78	0.81

The necessary useful section is 10.9 greater than above; but the difference is partly made up in the diminished amount of weakening. The formulas answer for many-shear rivetings for entire plates, δ being of value heretofore given.

In the riveting of plates we may put $F_n = F$, instead of as formerly

$= \frac{5}{4} F$. For the shearing resistances in the practically important directions are often found to be equal, and greater than the tensile strength in the direction of rolling. And the reasons in favor of $F_n < \frac{5}{4} F$ still hold while we have uniform distribution of the strain upon the rivets. Loss of friction-resistance in the case of plates is not to be feared; in many cases, because the joints would not be close without it, and riveting would be of no avail.

Making $F_n = F$; *i. e.*, the shearing = the tensile resistance we have

$$\text{for single riveting } D = \left(1 + \frac{\pi}{4}\,\frac{d}{\delta}\right) d \qquad (32a)$$

$$\text{double} \qquad D = \left(1 + \frac{\pi}{2}\,\frac{d}{\delta}\right) d \qquad (33a)$$

and for the ratio of useful to entire section,

$$\text{single riveting } \alpha = \frac{1}{1+\frac{4}{\pi}\frac{\delta}{d}} \qquad (34a)$$

$$\text{double} \quad \text{“} \quad \alpha = \frac{1}{1+\frac{2}{\pi}\frac{\delta}{d}} \qquad (35a)$$

e. g., if $\frac{d}{\delta} =$	1.5	2	2.5	3
for single, $\alpha =$	0.54	0.61	0.66	0.70
for double, $\alpha =$	0.70	0.76	0.80	0.82

For the case in which the strain is perpendicular to the direction of rolling, taking the tensile strength at $\frac{9}{10}$ths of that in the direction of rolling,

$$\text{for single riveting, } D = \left(1 + \frac{5\,\pi}{18}\frac{d}{\delta}\right) d$$

$$\alpha = \frac{1}{1 + \frac{18}{5\,\pi}\frac{\delta}{d}}$$

$$\text{for double riveting, } D = \left(1 + \frac{5\,\pi}{9}\frac{d}{\delta}\right) d$$

$$\alpha = \frac{1}{1 + \frac{9}{5\,\pi}\frac{\delta}{d}}$$

$$\text{Then for } \frac{d}{\delta} = 1.5 \qquad 2 \qquad 2.5 \qquad 3$$

$$\text{for single, } \alpha = 0.57 \quad 0.64 \quad 0.69 \quad 0.72$$

$$\text{for double, } \alpha = 0.72 \quad 0.78 \quad 0.81 \quad 0.84$$

Formulas (32*o*) and (35*a*) were used by Grashof and others.

§ 24.

Rivet-Pitch in Plate Girders.

A special treatment is necessary for plate girders. The distance between rivets in any row is to be determined. Connection with the vertical plate is effected exclusively by row I (Fig. 41). Without this the plate would slide freely between the two angle-irons; the row I must resist the greatest strain upon I caused by such sliding. Denoting the horizontal shearing force per unit of length upon I by H_I, and the permissible strain on a rivet by N, there are required for a unit of length, $n = \frac{\max. H_I}{N}$ rivets. If in a single row, the distance between rivets is

$$e_I = \frac{1}{n} = \frac{N}{\max. H_I} \qquad (37)$$

The rivets of the row I are double-shear, hence

$$N = 2 \frac{\pi d^2}{4} \frac{4}{5} b.$$

But this value is permissible only when the pressure upon the rivet-hole is not too great, which is always the case (§ 20) when $d > 3.2\delta$.

Since $\delta = \frac{1}{2}\mathrm{d}$, $d > 3.2\delta$; and if the pressure on the rivet-hole is not too great we may put

$$N = d\,\mathrm{d}\,2b, \text{ so that by (37)} \quad e_I = 2d\,\mathrm{d}\,\frac{b}{\max.\ H_I}$$

in which d is the thickness of the plate and the exact value of max. H_I is to be substituted.

As a hair-splitting calculation would be superfluous, we may proceed as follows: The horizontal shearing per unit of length is greatest in the neutral section; its value is $H_0 = \frac{V_X}{h}$; in which V_X is the total vertical shear in the section x, and h, the distance between the centres of tension and compression. From the neutral section to I, the horizontal shearing force diminishes but little; this may be provided for, if we put for h the larger value h_o, viz., the distance between the centres of gravity of the flanges; at I, $H_I = \frac{V_X}{h_o}$, approximately. The decrease is less, the thinner the vertical plate and the thicker the flanges; and with absence of vertical plate the equation is perfectly accurate. We have for I, e_I

$$= 2d\,\mathrm{d}\,h_o \frac{b}{\max.\ V_X}.$$

Formula (38) holds for constant as well as variable b. We assume the latter, and for wrought iron, according as in V_X, at x is always of the same or of alternating sign, as in (§ 17).

$$b = 700\left(1 + \tfrac{1}{2}\frac{\min.\ V_x}{\max.\ V_x}\right)$$

$$b = 700\left(1 - \tfrac{1}{2}\frac{\max.\ V_x'}{\max.\ V_x}\right)$$

If e_I varies with $\dfrac{b}{\text{max. } V_x}$ it is clear that in all practical cases this quotient, and therefore e_I, is the smallest at a support, both for simple and continuous girders. If e_I is to be constant for the whole length, the calculation should be made only for a support, and the first value of b is employed. However great the variation of e_I, by the old method of calculation, it is not so by the new, for towards the middle not only max. V_x, but also b diminishes. A constant value of e_I is desirable for small girders; for large, it may be made a little more at the middle of spans.

If a simple plate-beam is uniformly loaded, then, at supports, p being dead-load and q total load per unit of length.

$$\text{Max. } V_x = \tfrac{1}{2}\, q\, l; \text{ min. } V_x = \tfrac{1}{2}\, p\, l.$$

$$\therefore b = 700 \left(1 + \tfrac{1}{2} \frac{p}{q}\right)$$

and by substitution in (38) we obtain for the greatest permissible distance between rivets at the supports.

$$e = 2{,}800\ \mathrm{d}\ h_o \frac{1 + \dfrac{p}{2\,q}}{q\,l} \qquad (38a)$$

d and h_o being in centimeters.

(38) and (39) show that the pitch is proportional to the thickness of the vertical plate and the distance between the centres of flanges, which indicates how to increase the pitch if desired.

For *II*, Fig. (41), it is of import to know the number of horizontal plates. With more than one there is indirect transmission of force; since the strain is always taken up by the last plate imposed. If m is the number of plates imposed,

max. H_{II} the greatest horizontal shearing strain at II at the section x

$$n = m \frac{\text{max. } H_{II}}{N},$$

and for r rows,

$$e_{II} = \frac{r}{n} = \frac{r\,N}{\text{max. } H_{II}\, m}. \qquad (39)$$

Generally $r = 2$, and the rivets at II are one shear, and the condition $d =$ or < 3, $2\,\delta$ is always fulfilled, δ being the thickness of the horizontal angle-leg;

then $$N = \frac{\pi\, d^2}{4} \cdot \frac{4}{5}\, b,$$

and $$e_{II} = \frac{2\,\pi\, d^2}{5\, m} \cdot \frac{b}{\text{max. } H_{II}}.$$

The ratio of the horizontal shearing force at II to that at I is as the section of all the horizontal plates to the section of the entire flange; *i. e.*, if γ denote this ratio;

$$H_{II} = \gamma\, H_I = \gamma\, \frac{V_x}{h_o}$$

$$\therefore e_{II} = \pi\ {}^2 d h_o \cdot \frac{2}{5\, m\, \gamma} \cdot \frac{b}{\text{max. } V_x}. \qquad (40)$$

(38) and (40) give the ratio of theoretic pitches of II and I at any point x, and

$$\mathrm{e}_{II} = \frac{\pi\, \mathrm{d}}{5\, m\, \gamma\, \mathrm{d}} \cdot \mathrm{e}_I \qquad (41)$$

Suppose that at any point $m = 3$, $\gamma = \frac{3}{5}\, d = 3$ m.; $\mathrm{d} =$ 1 cm., then by (41), $e_{II} = 1.05\, e_I$.

Hence with ordinary ratios for one and two horizontal plates the pitch for *II* is more than for *I*; for three it is equal, and for more plates it is less. If, then, as often happens, the same pitch is employed for *I* and *II* (Fig. 42), it may be extended for 3 horizontal plates, without further calclulation of e.

Example.—Given simple plate-girders; $l = 7$ and 10 m. span; $h_o = 75$ and 110 cm.; $p = 900$, $q = 8{,}100$; $p = 1{,}000$, $q = 7{,}000$ kil. $\delta = 2.5$ cm. To determine e_1. From 38a

$$\text{for } l = 7 \qquad e_I = 2{,}800 \times 2.5 \times 75 \,.\, \frac{1 + \frac{1}{18}}{8{,}100.7} = 10 \text{ cm.}$$

$$\text{for } l = 10 \qquad {}_I = 2{,}800 \times 2.5 \times 110 \,.\, \frac{1 + \frac{1}{14}}{7{,}000.10}\ 12 \text{ cm.}$$

For loads not uniformly distributed use (38). We see that this method of calculation gives plausible values; while it is usual to assert that it gives values too great. If the permissible stress upon the rivet-walls had not been considered the value of e_I would have been greater by 0.63 d. But the pressure against the rivet-holes would then have been greater in the same ratio; and for $\delta = 1$ and $d = 3$ it would have been twice as large as the permissible value.

§ 25.

For framed trusses, the members of which bear only axial strain, there is no theoretical determination of rivet-distance; the formulas occasionally used are derived from erroneous hypothesis.

If a compressed flange between two vertices is composed of several adjacent pieces, rivets are necessary in order to bind these together, so that it may act as a whole piece. The resistance of bar to crippling is proportional to the minimum mo-

ment of inertia for an axis through the centre of gravity of a section: hence the resistance of the flange as a whole is much greater than the sum of the resistances of the separate parts.

For the same reason the flange should have a section with the greatest possible moment of inertia, while for tension flanges concentrated forms are generally to be preferred.

Theoretically, if the vertices are properly distributed, riveting of separate parts of a flange under tension is not necessary; still pieces lying close to one another are riveted in order to form tight joints.

In the flanges of trusses, for rivets of 2-3 cm. size, a distance of 14 to 20 cm. can be taken. In tension flanges they may be set further apart, especially if the pieces are thin, so that great force is not required to make a tight joint. The section should be weakened as little as possible by rivet-holes.

Suppose the web-members of a lattice-girder riveted to a vertex-plate, but separate from the flanges, then this corresponds to the vertical plate of plate-girders. The rivets by which the vertex-plates are fastened to the adjacent parts of the flange, have the same duty as the row *I* in plate-girders, Fig. 41; and the rivets by which the strain is transferred from the parts of the girder immediately adjacent to the other parts correspond to the row *II*. There is no transmission of any forces in the space between the joints of skeleton girders. In plate-girders, the transmission takes place in all sections, the vertices becoming infinitely close and continuous.

The line of the centres of gravity of the web-members should intersect in the gravity axis of the flange, because any excentric longitudinal strain of a piece causes a bending and unequal distribution of force. An attempt has been made to disprove this; but to do so it is assumed that generally, as with plate beams, the uppermost fibres of the compression-flange and the undermost of the tension-flange are more strained than those between, a hypothesis which theory

does not support. Formulas for distance of rivets based on the same hypothesis, and are therefore worthless. Under excentric transmission of forces, the flanges act like continuous girders, which are partly held fast at the supports.

The number of rivets binding the vertex-plate with the flange must be sufficient to transmit the resultant R of the stress in the web-members to the pieces of the flange intended to take up the strain R. In most cases there is only an indirect transmission (§ 21), and the necessary number of rivets, if there are m pieces between the vertex-plate and the piece to which R is to be transferred, is by (28)

$$n = (m + 1)\, n_1 = \frac{5\,(m+1)}{\pi\, d^2}\, \frac{\max.\, R}{b}$$

of which, at least $\frac{1}{m}$ must be assigned to the last mentioned piece.

In the section Fig. 43, *e. g.*, the size of the angle-irons is generally increased in proportion to the increasing strains in the flanges. R must pass to the angle-plate; and if $m=1$, because of an intervening plate, in the section Fig. 44, $m=1$; for though R is generally to be transmitted to the horizontal plate; yet it is transmitted only as far as the upper angle-irons, by the rivets which take hold of the joint-plate; and for further transmission from these to the horizontal plate, the rivets in the horizontal leg of the angle-iron are called into action; the number of these being determined by (42). Hence, in both cases the connection of the vertex-plate requires

$$n = \frac{10}{\pi d^2}\, \frac{\max\, R}{b} \qquad (43)$$

If the fraction $\frac{1}{v}$ of the resultant and the fraction $\frac{1}{w}$ pass into two adjacent plates, separated respectively by one and by

two plates from the vertex-plate, (which may happen with girders of constant chord-section, or at the points of support); then

$$n = n_1 + \frac{n_1}{v} + \frac{2\,n_1}{w} = \frac{5}{\pi d^2}\,\frac{\max.\,R}{b}\left(1 + \frac{1}{v} + \frac{2}{w}\right) \quad (44)$$

Only in very unfavorable cases the value of the quantity in the parenthesis amounts to 2; (generally $\frac{2}{w} = c$, because there is no transfer by two intermediate plates), so that formula (43) never gives too few rivets.

Denoting by α the angle between two pieces meeting at an angle and acting together (Fig. 45),

$$R = \sqrt{X^2 + Y^2 - 2\,X\,Y\,\cos\alpha}.$$

The maximum value of R should be known; but there is no formula to suit all trusses and all vertices; a value may be found, however, sometimes too large, sometimes exact, and never too small from the formula

$$\max.\,R = \sqrt{\max.\,X^2 + \max.\,Y^2 - 2\max.\,X\max.\,Y\cos\alpha}. \quad (45)$$

In the formulas for n, b has denoted the permissible tension per square cr. for the ratio $\frac{\min.\,R}{\max.\,R}$ or $\frac{\max.\,R'}{\max.\,R}$. But it is better to use the smaller of the two values. This is in some cases exact, sometimes too small, so that the value of n so found is large enough.

Example.—To determine the number of rivets required for the vertex-plate of the truss in Fig. 6, flange-forms as in Figs. 43, 44. $d = 2.5$ cm. $\alpha = 45°$.

$$\text{From (43).} \qquad n = 0.51\,\frac{\max.\,R}{b}.$$

For the vertex *IV V* substitution in (45) gives

$$\max R = 16{,}700 \text{ kil.}$$

For *IV*, $b = 758$; for *V*, $b = 742$.

$$\text{hence } n = 0.51 \frac{16{,}700}{742} = 12.$$

The upper vertices only were supposed to be loaded. To find max. R for the lower vertices more conveniently than by (45), recollect that diagonal and vertical abutting at an unloaded vertex suffer equal vertical strains, and reach maxima and minima values simultaneously. Hence at the vertices of the lower flange max. $R =$ max. $X \cos a$; and since $a = 45°$, max. $R =$ max. X.

The following are the results of the calculation :

Vertex	II,	III IV,	VI V,	V VI,
Max $R =$	21,200	21,000	16,700	15,625
$b =$	758	758	742	742
$n =$	14	14	12	11
Vertex	VI VII,	VII VIII,	VIII IX,	IX X.
Max $R =$	11,900	10,875	7,880	6,750
$b =$	688	688	531	531
$n =$	9	8	8	7

The number necessary for the securing of the web-members is not calculated. But n refers to the whole strain transmitted at the vertices. Hence, if, as in Figs. 43, 44, two vertex-plates are used, $\frac{1}{2}\, n$ rivets are required.

§ 26.

Riveting of Lattice Girders.

The preceding methods serve both for single and compound trusses. The use of continuous vertical plates instead of joint-plates is in general not judicious, the rivets being very

regularly affected; and the vertical-plates at the junctions of the web-members being disproportionately strained, because at these points the plate must receive its share of the stress on the flange, and the differently directed stress of the web-members. If the web-pieces are set at short intervals, as in lattice trusses, it is often convenient to substitute for the vertex-plates a continuous vertical-plate. But in that case, because of the unfavorable strain of the plate, induced by the vertex-plates, the whole plate should not be referred to the flange, and more rivets than usual should be employed in the fastening of the web-members.

The lattice-bars riveted to the vertical plates form a web corresponding to the vertical plate of plate-girders. The transmission to the flanges of the force, received by the vertical plates from the truss bars, takes place no longer at distinct vertices, but in a continuous way; so that rivet distances are determinate.

The rivet pitch of I, Fig. 47, if the flanges are parallel, is obtained like that of plate-girders, by (37); but in this case,

$$\therefore N = 2\frac{\pi d^2}{4} . \frac{4}{5} b$$

$$\text{so that } e_I = \frac{2}{5}\pi\, d^2\, h_o . \frac{b}{\text{max. } V_x} \qquad (46)$$

The value of $\dfrac{b}{\text{max. } V_x}$ is (as with continuous girders) least at a point of support; and hence for the least distance, and because V_x is of the same sign at all points, for the least rivet-pitch,

$$b = 700\left(1 + \frac{1}{2}\frac{\text{min. } V_x}{\text{max. } V_x}\right)$$

Towards the middle of the truss or of a span, V_x may have a different sign, so that if the rivet-pitch is to vary,

$$b = 700\left(1 - \frac{1}{2}\frac{V'_x}{V_x}\right) \text{ may be used.}$$

For simple lattice-girders, with uniformly distributed weight of structure p, and total load q, we have for the least rivet-pitch the same value of max. V_x and b as those obtained in

§ 24 ; hence $e_{II} = 560\ \pi\, d^2\, h_o \dfrac{1 + \frac{p}{2q}}{q\, l}$ (46_a) in which d, h_o and e_I are in centimeters.

For the rivet-pitch in I, for two rows as in Fig. 47,

$$e_{II} = \pi\, d^2\, h_o \frac{2}{5\,m\gamma}\,\frac{b}{\text{max. } V_x} \qquad (47)$$

and
$$e_{II} = \frac{e_I}{m\gamma}. \qquad (48)$$

in which m is the number of horizontal plates, and γ the ratio of their total section to that of the whole girder.

Equations (46) and (47) are specially applicable to horizontal flanges. If the flange is inclined at any point, the force transmitted is $\dfrac{1}{\cos a}$ times greater; and the corresponding rivet-pitch $\cos a$ times less than for the horizontal.

If the flange is of such form that two rows of one-shear rivets are required for I instead of a single row of two-shear, the formulas for e_I would be the same; if two rows of two-shear or four of one-shear are used, then the value of e_I should be

twice as large; so the rivet-pitch for II can be determined without recourse to the general formulas (37), (39).

Example.—For bridges of spans stated, loaded by heavy locomotives, Schwedler puts $p = 800 + 30l$, and thence deduces the values of q; parallel flanges.—To find the minimum pitch for a row of two-shear rivets, Fig. (47) or two rows of one-shear; h_0 being $\frac{1}{10}\, l$. $d = 2.5$ cm.

Substitution in (46a) gives the following figures:

$l =$	8	10	15	20	30	40	50 m.
$p =$	1,040	1,100	1,250	1,400	1,700	2,000	2,300 kil.
$q =$	9,400	8,400	7,050	7,000	6,900	7,100	7,200
$e_I =$	12.3	13.9	16.9	17.3	17.9	17.7	17.7

For such large values make e_I constant along the girder.

§ 27.

Connections at Joints.

The disposition of rivets at joints may differ in cases of tension and thrust upon the abutting piece. For in the first case the plate only takes up the strain, while in the second the transmission may be regarded as direct. This would happen only with perfect contact at joints, which cannot be assumed, and is not under control. Since the splice-plate must resist lateral shocks and disturbance it is the ordinary practice to calculate for the fastenings at joints of members under compression, just as in the case of tension; only when the constructive relations seem to require it, is any diminution made in the length of the splice-plate or the number of rivets.

Double splice-plates are to be preferred to single. If a single splice-plate is used for a piece under tension, a couple tends to bend it and to spring the rivet heads. Single plates for a piece under compression induce danger of crippling at the joint. These points are of import only for single riveting; and in many cases, *e. g.*, in that of a flange for which

a single splice-plate is necessary, because the other side of the bar is not free; the adjacent bar preventing bending or crippling. In the latter case it is much worse that a part of the stress should probably pass not to the splice-plate but into the abutting rod. At the time of the building of the Britannia Bridge it was found, by experiment, that pieces so disposed bore about $\frac{3}{2}$ of the intended stress.

As the splice-plates must receive a strain, as great as that upon the piece, its useful section must be the same. Generally double plates need be only half as thick, requiring only half as many rivets as single, because they are two-shear. The necessary length of splice-plate diminishes with the necessary number of rivets, so that double plates are economical. And the strain from the rod is distributed more uniformly among the rivets, the less the number of rivets in the direction of the strain. In general double plates are the best.

The stress on an abutting piece is communicated to the splice-plate by the rivets. Since the rivets on each side of the joint have to bear the same shearing strain as the strain upon the rod, whatever its nature; the number of rivets necessary each side of the joint (Fig. 48, 51) is

$$\text{for single plate, } n_1 = \frac{5}{\pi d_2} . F \qquad (26)$$

$$\text{for double plate, } n_2 = \frac{5}{2 \pi d^2} . F \qquad (27)$$

These formulas are applicable only when the splice-plates are in direct contact with the piece. It must not be forgotten that, if there is between the two an intervening plate, the splice-plate must have twice as many rivets, and must, therefore, be twice as long. This is a case of indirect transmission of force, and what was said in § 21 applies. It is obvious that plate *II* must first take up the strain of *I*, and that the piece

a a serves as splice-plate for *I*. That this may be possible without over-strain, *II* beyond *a a* must have first been released from the strain, for which *n* rivets are necessary on each side. Two plates often joint at the same place. If the joints are disposed as in Fig. 53, two plates must be set, of the thickness of the plate, and upon each side *n* rivets. But usually the joints break; then the splice-plates need be but half as thick as the plates, and $2n_1 + 2n_2 = 3n_1$ rivets are required; shown in Fig. 54; in which the full line corresponds to the whole, the dotted line to one-half of the strain *P*. Lengthening of the splice-plate (Figs. 54, 55) is not necessary, though it is sometimes done for convenience.

The necessary number on each side being determined, the minimum distance along and across the line of direction of the force from the edge can be determined by the method of § 22. Care must be taken that the piece be not weakened at the joint. In the flanges this can always be done; the net-section of the rod through the outermost rivet-row *I* of the splice-plate (Fig. 56) must not be less than that outside of the connecting joint; and the number of rivets must increase per row by, at the most, as many as stand in row *I* (§ 22). The net-section of the splice-plate between the innermost rows must be counted as its useful section; if this is diminished its thickness must be increased.

At the joints of the vertical plates of plate-girders the effect of weakening is not of so much account; it is not the practice to joint where the vertical shearing force and the moment are both of large amount at the same time; at other points the plate is always stronger than necessary (§ 17). Still the number of rivets should be adjusted to the section of the vertical plate, because a uniform distribution of strain upon it is not to be expected.

As the rivets are double shear

$$n = \frac{5}{2\,\pi\,d_2}\,F$$

This would give for strain on one rivet

$$N = 2 \frac{\pi d^2}{4} \cdot \frac{4}{5} b$$

which for thin plate gives too much strain on the holes.

Therefore make

$$N' = 2\, d\, \delta\, b.$$

hence

$$n = \frac{N'}{N} n_2 = \frac{F}{2\, d\, \delta}$$

$$n = \frac{h}{2\, d}$$

After the most careful disposition of connections at joints, there may still be weak points. Hence pieces which form any portion of the structure, as a flange, should never meet at the same place; and the joints should be distributed as uniformly as possible.

APPENDIX.

§ 28.

The Methods of Gerber, Muller and Schäffer.

The first work based upon Wöhler's results was published in 1872; and was adopted by the Bavarian Government as a "Programme for the Calculations of Iron Constructions."

His fundamental notions were as follows: Suppose a bar of unit section broken by a resting load. The same effect can be caused by a strain in part constant (c); increasing repeatedly and very often to a value d. Hence, the difference in strain is equal to a value $\tau\, d$, and

$$c + \tau\, d = t = \sigma\, d,$$

σ being a co-efficient determined by the conditions.

If B_c is the continued strain, and the passing strain B_v, the resultant can be reduced to a dead-load by the equation.

$$B_c + \tau\, B_v = B_r = \sigma\, B_v\,,$$

and $$F = \frac{B_r}{b_r}$$

in which b_r is the permissible strain per sq. unit for resting-load.

Gerber calls B_r the reduced force. The values of τ and σ can be found for special values of c, d and t, given by Wöhler. The ratio varies with

$$\varphi = \frac{c}{d} = \frac{B_c}{B_v}\,.$$

To obtain the law of this variation, Gerber puts

$$x = \frac{c}{t}, y = \frac{d}{t}$$

and represents the relations between x and y by a parabolic curve. This process determines relations from which the values of σ, τ, corresponding to φ can be found.

Gerber's formulas also serve for alternating tension and compression. B_c and B_v receive opposite signs, so that φ may be positive or negative. By B_c is signified only the strain of constant load, by B_v that of live load. If $B_c = -B_v$; $\varphi = -1$.

Gerber gives a table of co-efficients σ for iron of values from $\varphi = 0$ to $+8{,}720$ and $-9{,}720$. In the table they all have the positive sign; but it must be remembered that B_r may not have the same sign as B_r, but must take that of $B_c + B_v$.

In the application, so as to allow for impact, making max. B_v 1.5 times the actual live-load, we have

$$\varphi = \frac{B_c}{\text{max. } B^v} \cdot \qquad \text{(A)}$$

Find the σ corresponding to this φ in the table, hence

$$B_r = \sigma \text{ max. } B_v \qquad \text{(B)}$$

then

$$F = \frac{B_r}{b_r} \cdot \qquad \text{(C)}$$

For permanent bridge structures, Gerber makes $b_r = 1{,}600$; for light structures, $b_r = 2{,}400$.

The total strain $B_c + B_v$ may be positive as well as negative; B_v is 1.5 times the live-load; φ, B_r and F must be calculated for both limiting values of B_v; B_v and F taking opposite signs, and the sum of both numerical values of F giving the required section.

If p is the dead-load and z is 1.5 times the live-load per running meter,

$$\varphi = \frac{B_c}{\max.\ B_v} = \frac{p}{z}$$

and the actual strain of a flange member is

$$B_c + \frac{2}{3} \max.\ B_v ,$$

hence the permissible strain per square centimeter

$$\frac{B_c + \frac{2}{3} \max.\ B_c}{F}$$

$$= \frac{B_c + \frac{2}{3} \max.\ B_v}{\sigma \max B_v} = 1{,}600$$

$$= \frac{\frac{2}{3} + \varphi}{\sigma} . 1{,}600$$

This gave the figures in the first table of § 30.*

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* Gerber's publication does not contain the special application to flanges. The formulas are given here for the sake of comparison in § 30, and to show our view of some doubtful points in his treatment. Having said that max. B_v is to be calculated for 1.5 times the live-load, he adds that this value multiplied by τ gives the variable stress as a function of the permanent strain. What τ? we ask. Naturally the τ for the corresponding ϕ. But is ϕ to be calculated for live-load, or for 1.5 times that quantity? Take it for the latter,

$$b = \frac{1 + \phi}{\phi + 1.5\,\tau}\ 1600$$

Schäffer puts $x = \frac{d}{t}$; $y = \frac{c}{t}$; finds the relation between them in the same way as Gerber, but differs essentially in finding the dimensions.

This is his process. Let max. B be the greatest total strain; min. B the least of the same sign, or the greatest of opposite sign, then, putting $c+d=a$,

$$\frac{d}{a} = \frac{\text{max. } B - \text{min. } B}{\text{max. } B}$$

in which both limiting values are to be substituted with signs, so that $\frac{d}{a}$ is always positive.

Let max. B_v, min. B_v refer to live-load, B_c to dead-load; then, in most cases,

$$\text{max. } B = B_c + \text{max. } B_v$$
$$\text{min. } B = B_c + \text{min. } B_v$$

Min. B_v is either zero, or of opposite sign to max B_v;

then
$$\frac{d}{a} = \frac{\text{max. } B_v - \text{min. } B_v}{B_c + \text{max. } B_v}$$

and with these results, the value of the working resistance (a) can be found.

If the section is taken so that the strain per square unit is a, fracture may just take place. Schäffer now attempts to attain

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With this formula, I have, by way of experiment, calculated the values for the first table in § 30, and found that they are equal at the extremes, and for other cases they differ by at most 20 kil.

Gerber says, with regard to members under alternating compression and tension: " From these reduced forces, signs being regarded, the dimensions may be found." We hope that the above will suffice for this.

the requisite security by regarding the effect of live-load throughout as n-fold.

The section is calculated as follows:

By moments, $$\psi = \frac{n\,(\text{max. } B_v - \text{min. } B_v)}{B_c + n \text{ max. } B_v} \qquad (A)$$

If ψ is known, the relation assumed between x and y gives a fictitious working resistance (for ψ is not the actual ratio $\frac{d}{a}$).

$$a = \frac{-3\,\psi + \sqrt{13\,\psi^2 - 16\,\psi + 16}}{(2-\psi)^2}\,t \qquad (B)$$

$$\text{hence } F = \frac{B_c + n \text{ max. } B_v}{a} \qquad (C)$$

The greatest unit-strain is

$$b = \frac{B_c + \text{max. } B_v}{F} = \frac{B_c + n \text{ max. } B_v}{B_c + n \text{ max. } B_v} \cdot \ a$$

For flanges of girders $p = \frac{r+z}{p+nz}\,a$, p and z being respectively dead and live-load.

Schäffer makes $n = 3.5$ or 4. In § 30 the first value is used.

Müller starts with the assumption that every strain beyond the elastic limit often repeated must cause rupture. The original strength u is the least strain in one direction sufficient to cause rupture, and is identical with the ordinary limit of elasticity. For smaller differences of stress or greater value of $\frac{c}{d}$ rupture is first possible under the strain a (the working-strength); hence, there is an indefinite number of elastic limits, varying from u to t.

Laying off Wöhler's values of c as abscissas (initial com-

pressions negative), and the values of a as ordinates, we obtain a curve shown in Fig. 57. Müller prolongs to the c-axis, by which he determines by "analogy" (how, it does not exactly appear), the value of the original resistance to compression, and so completes Wöhler's results.

From this curve for any given ratio

$$\varphi = \frac{c}{d} = \frac{B_c}{\text{max. } B_v}$$

the value of a can be found, and that of b by introduction of a safety factor. Müller employs the factor $\frac{1}{3}$; and in this includes the effects of temperature and of corrosion. He considers that the effect of increase of temperature is the same as that of a strain producing an equal elongation. It is admitted that the effects of temperature and stress cannot well be added; still both take part in the "wear;" and this compels the reduction of the absolutely greater strain, under greater permanent load, because with the presence of other stresses the danger of reaching the breaking limit is increased.

So the value $\beta = \frac{a}{u}$ is modified, for reasons assigned that are not quite clear, and which are really unsound, judging by results; and a series of values of φ determined by the completed curve of Wöhler's results, giving $b = \frac{1}{3} \beta u$. Müller makes $u = 1{,}600$, and gives two tables of permissible strains; one for tension only, the other for alternating tension and compression. It is not recommended to use them.

§ 29.

Remarks.

The amount of permissible strain depends upon a general, but necessarily approximate determination of the relation between a and the differences in strain. Wöhler's law is the

only point of departure, since it alone is of general application, and correct in all cases.

The special results which he obtained must be employed in calculation of permissible strains; but with discrimination and without accepting them as final; just as was done with the results of former experiments. Safety co-efficients have always been, and must always be, employed.

Considering that Wöhler's law has not been disputed by any one, and that it has been recently confirmed by Spangenberg's experiments, it is surprising that a method of calculation, always conceded to be false and dangerous, should be offically tolerated. Further experiments are desirable; but they would only have a significance like that which a new method of testing tensile strength formerly had; neither the law nor the general formulas deduced from it would be altered, We know already that each material gives its own figures; and it is difficult to see, for what special tests we must wait before adopting the law. For tests on special bridge material? According to all previous tests it must not be assumed that smaller differences occur with *one* kind of metal, than between different kinds, of well known qualities (§§ 5, 12); and the iron tested by Wöhler, at least, was not better than that required for good bridges (§ 13). Or for tests of kinds now in the market? Can these be had to-morrow? No sufficient reason for not introducing a new method of calculation has been given thus far; and we need not notice those whose judgment is hindered by their conservatism. Or, shall we wait till the new method comes from abroad? The question is not about theoretical crotchets; and the names of the experienced practical men who have made these investigations should be sufficient warrant to those who are without experience. Call for as many co-efficients of safety as you will, the old method is no longer tenable.

There remains the question of choice between the different proposed methods. If a new process is to be generally

accepted, it must be theoretically sound, simple in application, and not contradictory to past experience. It is not finally proven that the working resistance a varies in every kind of material, according to the same general law; but, it may be demanded that actual departures opposed to the general formulas, be confined within the limits of the differences of kinds recognized as good. In all these respects Launhardt's process is to be preferred. This is *prima facie* true, and conviction comes with more exact testing and comparison of practical calculations. It was after such comparison that we felt that it was necessary to complete and extend his method.

Launhardt's formula in form (3) is the expression of Wöhler's law. It determines the limiting values of α, and the only arbitrary element is the choice of the interpolation formula for α

This choice is confirmed by Wöhler's experiments, adapted to testing only, and besides other by experiments with iron. Even for more exact results than those which concern us here, they are sufficient. Further hypothesis and more complex developments would be superfluous.

Gerber's theory (§ 28) is clear; but the relation between a and $\frac{c}{d}$ is determined in an artificial way. This was necessary in order that the formula might hold for alternation of tension and compression; but the application in this case is not simple. Not only must (A), (B), (C) be employed twice, but the whole previous method of static calculation must be changed. The effect of dead-load, and the positive and negative maximum effects of the live-load must be separately determined for each piece. This is also true of Schäffer's method, which also takes too much account of passing load, and none at all of fixed. His formulas give the security less the greater the fixed load; and when this load only is considered, they give as permissible strain the total ultimate strength $t = 3{,}500$.

(§ 28; for $B_v = o$; $\psi = o$, $a = t$ and $b = a$.) Gerber for dead-load puts $b = 1{,}600$, which is not exact. Since Schäffer does not provide safety against fixed load, it happens that sometimes in case of alternating tension and compression a greater permissible value is found than for tension alone, viz: whenever

$$600 < \frac{B_c + \text{max. } B_v}{B_c + \text{max. } B_v} a$$

$$i.\ e., \text{ for } B_c > \frac{600\, n - a}{a - 600} \text{ max. } B_v$$

This happened in one case of the second table of § 30. When tension and compression are equal (B) gives an indeterminate value.

Müller's process depends in part upon untenable assumptions. It is not certain that a single increase of temperature has the same effect as a single load: previous observations are to the contrary (§ 10). We have seen that at temperature from 100° to 200° C, a greater load is borne than at ordinary temperature, though both effects combine. Even if Müller's hypothesis were correct, it could not be applied in all cases. For by it the permissible stress is greater for alternate tension and compression than for tension only and restoration.

The methods of Gerber, Schäffer and Müller agree in this that they are based too exclusively on Wöhler's results. The last, stripped of unnecessary incidentals is the graphic representation of Wöhler's numerical results; and Gerber's and Schäffer's methods can be made useful only by calculation of tables which must depend on Wöhler's tests; and it is a question whether the old tables would be of avail; but formulas I and II are independent of special values, and can be accommodated to new results, while safety-factors can be taken at will.

Launhardt's view of the effect of impact upon the calculation

of permissible stress seems not adequate. He deduces from Wöhler's tests the working strength of iron, not considering impact,

$$a = 2{,}190\left(1 + \frac{5}{6}\,\frac{\text{min. } B}{\text{max. } B}\right)$$

The effects, not local, of impact consist in the most unfavorable case of the increase of max. B, which determines the sectional area, and the diminution of $\frac{\text{min. } B}{\text{max. } B}$ which diminishes the value of the working-strength. Launhardt takes into account only the latter by putting

$$a = 2{,}190\left(1 + \frac{1}{2}\,\frac{\text{min. } B}{\text{max. } B}\right)$$

In this way impact is more regarded the greater the value min. B, and is left out of account when min. $B = o$, as often happens. This cannot be meant. Regarding only tension or compression only, a cannot be less than $u = 2{,}190$; but in a bar alternately pulled and let go partially or wholly, impact causes in the most unfavorable case alternating tension and compression, so that the working strength is found by formula *II*. Let $A\ B$, Fig. 58, be the curve of working-strength, without reference to impacts, then, according to Launhardt, when these are considered, $C\ D\ E$ would take the place of $A\ B$, and when all are taken into account, $F\ C$ would take the place of $A\ B$. With this, Launhardt would probably agree.

§ 30.

Comparisons.

The following table contains the permissible strains b, calculated by the new methods for construction-members under

tension or compression only, and for which the ratio of limiting strains $\frac{\text{min. } B}{\text{max. } B}$ has the value assigned.

For flanges of girders, $\varphi = \frac{p}{q}$; in which p = dead-load and $q = p + z$, the total load per unit of length.

For comparison the numbers are given which are found by the formula $b = \frac{p + z}{p + 3z}\, 1{,}600$, as applied by Gerber in the calculations for the Mayence bridge.

In the 6th column are the figures corresponding to Launhardt's formula.

$$b = 800\,\left(1 + \frac{1}{2}\frac{\text{min. } B}{\text{max. } B}\right)$$

$\varphi = \frac{p}{q}$	$\frac{p}{z}$	Mayence Br.	Gerber.	Schäffer.	Launhardt.	Formula (11).
0	.0	533	646	600	800	700
$\frac{1}{6}$	0.2	600	740	712	867	758
$\frac{2}{7}$	0.4	659	820	814	914	800
$\frac{3}{8}$	0.6	711	889	910	950	831
$\frac{4}{9}$	0.8	758	947	1,000	978	855
$\frac{5}{10}$	1.0	800	997	1,088	1,000	875
$\frac{6}{11}$	1.2	838	1,043	1,171	1,018	891
$\frac{7}{12}$	1.4	873	1,080	1,250	1,033	904
$\frac{2}{3}$	2.0	960	1,172	(1,640)	1,066	933
1	∞	1,600	1,600	(3,500)	1,200	1,050

It is seen that Schäffer's formulas give too large security for permanent load when small, and too small when large.

For changes in tension and compression the permissible strains by Gerber's and Schäffer's methods, being dependent

not always alone upon φ, but also upon B_o, concrete cases must be used for comparison.

The figures in the following table under II, III, IV, are the strains given in Ritter's Dach. and Brück. Constr. for 3 diagonals of a truss. The strains per square cr. as determined by the American method are found by the formulas

$$b = \frac{\text{max. } B}{F}$$

$$F = \frac{\text{max. } B + \text{max. } B'}{700}$$

$$b = \frac{\text{max. } B}{\text{max. } B + \text{max. } B'} .700$$

The results are opposite "America."

	I.	II.	III.	IV.	V.
Max. B....	arbitrary.	+ 15,380	+ 6,230	+ 9,550	arb.
Max. B'....	0	− 530	− 1,280	− 4,600	arb.
B_c	0	+ 2,120	+ 710	+ 710	0
$\frac{\text{Max. } B'}{\text{Max. } B}$	0	0.034	0.206	0.565	1
Gerber ...	646	574	512	437	380
Schäffer...	600	(609)	542	436	334
Form. (12).	700	688	628	502	350
America..	700	677	581	472	350
Europe....	700	700	700	700	700

If any one refuses to avail himself of the benefit of the new method as regards economy of material, it is his own affair. The structure will not be the safer for this refusal; and whoever squanders material must answer to the one who has to pay for

it. But it cannot be assumed without risk, that permanent pieces under alternate tension and compression may be subjected to a stress of 700 kil. per square cr. If one does no more, let him at least use the American formula (p. 7), for the sections of pieces alternately under tension and compression, which gives results fairly agreeing with those obtained according to Wöhler's law from formula (12).

Fig. 9.

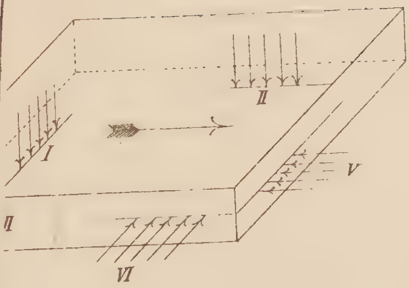

Fig. 10.